

## Learning Design about Proving Trigonometry Identity Using Problem-Based Learning (PBL)

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**Abstract:** This study aims to develop the design of learning proof of identity trigonometry in grade 10 Math and Science using PBL. Learning design is compiled using design research according to Gravemeijer & Cobb (in Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006), focusing on the first stage, namely: preparing for the experiment. The results of the study were in the form of a trigonometric identity proof learning design using PBL which had been tested on 33 Math and Science students of grade 10 in one of the private high schools in Yogyakarta. The learning process is adjusted to the syntax of PBL according to Ibrahim and Nur (Rusman, 2016). Phase 1: Orientation of students to the problem. Educators convey material topics, apperception, and explain in general the processes that will be experienced by students. Phase 2: Organize students to learn. Educators divide students into groups to work on Student Worksheet 1. Phase 3: Guiding individual or group experiences. Educators go around to each group to provide support in solving problems. Phase 4: Develop and present the work. Educators ask students to collect Student Worksheet 1 answers and then ask two students to present their group answers in front of the class. Phase 5: Analyze and evaluate the problem-solving process. Educators guide the analysis and evaluation process in accordance with the learning objectives to be achieved.

**Keywords:** learning design, trigonometric identity, problem based-learning syntax

### Introduction

Based on the experience of the researcher teaching proof of trigonometric identity, this topic is not an easy topic to learn by students. In the 2016-2017 school year, researchers taught proof of trigonometric identity using the lecture method. By using this method, the learning outcomes that students get are not satisfactory. From 35 Math and Science students of grade 10-6 and 34 Math and Science students of grade 10-7 in the academic year, data obtained 54% of students in grade 10-6 were not completed and 82% of students of grade 10-7 were not completed. The average grade 10-6 bill is 71 and grade 10-7 is 64. The data makes researchers interested in composing learning designs about proving trigonometric identity by using problem-based mathematical learning models.

Based on the background described above, the researcher proposed the following problems: How is the design of learning to prove the identity of trigonometry in grade 10 Math and Science using PBL? The purpose of this study was to develop a learning design proof of identity trigonometry in grade 10 Math and Science using PBL.

### *Design Research and Problem Based Learning*

Gravemeijer & Van Erder (Prahmana, 2017: 13) states that design research is a research method that aims to develop a local instruction theory (LIT) with cooperation between researchers and educators to improve the quality of learning. According to Prahmana (2017: 15) there are two important aspects related to design research, namely hypothetical learning trajectory (HLT) and local instruction theory (LIT). HLT is a hypothesis or prediction of how students' thinking and understanding develop in a learning activity (Prahmana, 2017: 11). Broadly speaking, LIT is the final product of HLT that has been designed, implemented, and analyzed the learning outcomes (Prahmana, 2017: 21). According to Gravemeijer & Cobb (in Van den Akker, Gravemeijer, McKenney, and Nieveen, 2006), design research consists of three stages, namely: preparing for the experiment, experiment design, and retrospective analysis.

Problem Based Learning (PBL) is one of the learning models that was first introduced in the 1970s at the Faculty of Medicine, McMaster University, Canada as an effort to find solutions in diagnosis by



making questions according to the situation (Rusman, 2016). Linda Torp and Sara Sage (2002) state their understanding of PBL as follows:

*Problem-based learning is focused, experiential learning (minds-on, hands-on) organized around the investigation and resolution of messy, real-world problem. PBL—which incorporates two complementary processes, curriculum organization and instructional strategy—includes three main characteristic, namely (1) engages students as stakeholders in a problem situation, (2) organizes curriculum around a given holistic problem, enabling students learning in relevant and connected ways, and (3) creates a learning environment in which teachers coach student thinking and guide student inquiry, facilitating deeper levels of understanding.*

Ibrahim and Nur (in Rusman, 2016) suggest that PBL is one of the learning approaches used to stimulate high-level thinking of students in situations that are oriented to real-world problems, including learning how to learn.

From the opinions of some experts, researchers define PBL as a learning approach that presents contextual problems so as to stimulate the interest of students to learn. Students work in teams to solve the contextual problems given by educators. PBL is a learning method that challenges students to “learn how to learn”, working in groups to find solutions to real-world problems given by educators. Contextual issues given by education are deliberately chosen to foster curiosity and high interest in finding solutions.

The principles or characteristics of PBL (Rusman, 2016) are (1) problems become the starting point in learning; (2) the problems chosen are problems that exist in the real world and are not structured; (3) problems require multiple perspectives (multiple perspective) in solving them; (4) problems can challenge the knowledge, attitudes, and competencies of students and then students can identify learning needs and new fields of learning; (5) learning self-direction becomes the main thing; (6) the use of diverse sources of knowledge, their use, and evaluation of information sources is an essential process in PBL; (7) learning is collaborative, communicative, and cooperative, (8) developing inquiry and problem solving skills as important as mastering the content of knowledge to find solutions to a problem; (9) openness of processes in PBL includes the synthesis and integration of a learning process; and (10) involves the process of evaluating and reviewing the experiences and learning processes of students. Ibrahim and Nur (Rusman, 2016) propose PBL steps as follows.

Table 1. PBL Syntax According to Ibrahim and Nur (Rusman, 2016)

Phase	Indicator	Educator's Behavior
1	Student orientation on the problem	Explain the purpose of learning, explain the logistics needed, and motivate students to be involved in problem solving activities.
2	Organizing students to learn	Helping students define and organize learning tasks related to the problem.
3	Guiding individual or group experiences	Encourage students to gather useful and appropriate information, carry out experiments to get explanations and problem solving.
4	Develop and present the work	Helping students in planning and preparing suitable works such as reports, and helping students to share assignments with other friends.
5	Analyze and evaluate the problem solving process	Helping students to reflect and evaluate the investigations and processes that students use.

## Research Methods

The type of research used is design research according to Gravemeijer & Cobb (in Van den Akker, Gravemeijer, McKenney, and Nieveen, 2006) focusing on the first stage, namely: preparing for the experiment. This research was conducted in March-April 2018.

The primary data source of this study is the HLT Trigonometric Identity topic for class X Math and Science in which there is a draft Student Worksheet (SW).

## Results and Discussion

Learning objectives such as those listed in HLT are: (1) students can prove trigonometric identity and (2) students can find strategies to prove trigonometric identity. This section will show the design of learning in accordance with the syntax of problem-based learning according to Ibrahim and Nur and adapted to the learning objectives above.

### Phase 1:

Educators deliver material topics to be studied, after which educators explain in general the process to be undertaken, namely educators will provide problems and students will be divided into groups to solve these problems.

After explaining the process to be undertaken, educators provide apperception by asking: What are the basic identities that have been learned in previous learning?

If students can mention the basic identity that has been learned in the previous learning process, educators give appreciation and then continue in the next phase. If students are unable to mention the basic identity that has been learned in the previous learning process, educators guide students to recall the basic identity that has been learned.

### Phase 2:

Educators divide students into groups while dividing the SW. Group members have been written on the SW. Educators divide students into groups based on mathematical achievements on previous topics.

<p>Problem:</p> <p>Prove the following trigonometric identity:</p> $\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$
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Next educator asks students to work in groups. Educators convey rules that must be done in groups, namely the process of discussion is only carried out in groups and may not discuss between groups. If there are questions, the group is asked to ask the educator directly. Educators provide opportunities for students in groups to ask about the rules of the game that have been set by educators. If there are questions, the educator answers the questions given, if there are no questions, the educator invites students in the group to start group discussions. The educator asks students to look at the problems listed in the SW, then ask the students to recall the basic identity mentioned earlier. Learning because it is important to be able to solve the problems given.

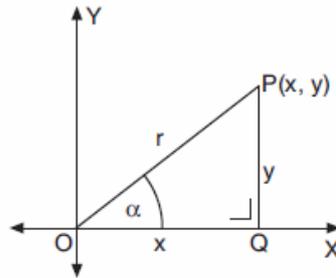
### Phase 3:

Educators go around to each group to provide support in solving problems.

*Alternative Answers 1.* If while traveling around the group have not found a way to solve the problem, then the educator encourages and gives support by inviting students to re-read the basic identity contained in the notes. After students understand the basic identity again, educators ask, which tribe can be replaced by using the basic identity formula. Can  $\tan \alpha$  be replaced with another? Can  $\sin \alpha$  be

replaced with another? Can  $\cos \alpha$  be replaced with another? Can  $\cot \alpha$  be replaced with another? If it's been replaced, what else can a mathematical process do? Educators provide opportunities for students to discuss the process to resolve the problem again.

*Alternative Answers 2.* The group answers but uses a right triangle assistance strategy. For example, the answer to the group is as follows:



$$\frac{1 - 2\cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

$$\frac{1 - 2\left(\frac{x}{r}\right)^2}{\frac{y}{r} \cdot \frac{x}{r}} = \frac{y}{x} - \frac{x}{y}$$

$$\frac{1 - \frac{2x^2}{r^2}}{\frac{yx}{r^2}} = \frac{y^2 - x^2}{xy}$$

$$\frac{r^2 - 2x^2}{r^2} = \frac{y^2 - x^2}{xy}$$

$$\frac{r^2 - 2x^2}{yx} = \frac{y^2 - x^2}{xy}$$

$$\frac{x^2 + y^2 - 2x^2}{yx} = \frac{y^2 - x^2}{xy}$$

$$\frac{y^2 - x^2}{yx} = \frac{y^2 - x^2}{xy}$$

*proven*

The educator appreciates the effort that has been made by the group and asks the group to think of other ways by giving a support in the form of questions what if not using a help triangle? Which tribe in the left or right side can be replaced by using the basic identity that has been learned? Educators give groups opportunities to discuss again.

*Alternative Answers 3.* Groups prove by using one special angle, for example:

$$\frac{1 - 2 \cos^2 60^\circ}{\sin 60^\circ \cos 60^\circ} = \tan 60^\circ - \cot 60^\circ$$

$$\frac{1 - 2 \cdot \left(\frac{1}{2}\right)^2}{\frac{1}{2} \sqrt{3} \cdot \frac{1}{2}} = \sqrt{3} - \frac{1}{3} \sqrt{3}$$

$$\frac{1 - \frac{1}{2}}{\frac{1}{4} \sqrt{3}} = \frac{2}{3} \sqrt{3}$$

$$\frac{\frac{1}{2}}{\frac{1}{4} \sqrt{3}} = \frac{2}{3} \sqrt{3}$$

$$\frac{2}{\sqrt{3}} = \frac{2}{3} \sqrt{3}$$

$$\frac{2}{3} \sqrt{3} = \frac{2}{3} \sqrt{3}$$

*proven*

Educators give appreciation then educators ask: If  $\alpha$  on the question is replaced with  $60^\circ$  the proven identity formula. How about  $30^\circ$ ? Does it still meet? What about  $45^\circ$ ? Does it still meet? What about  $569^\circ$ ? How much can we prove identity using numbers? Can we prove trigonometric identity with numbers? Educators invite students in groups to make conclusions about proving using numbers.

*Alternative Answers 4.* The group answers by describing one side but the other side is written continuously. For example, the answer from the group deciphers the left side and writes continuously the right side:

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

$$\frac{\sin^2 \alpha + \cos^2 \alpha - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

$$\frac{\sin^2 \alpha - \cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

$$\frac{\sin^2 \alpha}{\sin \alpha \cos \alpha} - \frac{\cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

$$\frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha} = \tan \alpha - \cot \alpha$$

$$\frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha} = \tan \alpha - \cot \alpha$$

$$\tan \alpha - \cot \alpha = \tan \alpha - \cot \alpha$$

*proven*

Or, the group decipheres the right side and writes down the left side continuously:

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha}$$

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \frac{\sin^2 \alpha - \cos^2 \alpha}{\sin \alpha \cos \alpha}$$

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \frac{1 - \cos^2 \alpha - \cos^2 \alpha}{\sin \alpha \cos \alpha}$$

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha}$$

*proven*

Educators appreciate the efforts that have been made by the group and ask the group to discuss it again is there another way or alternative?

*Alternative Answers 5.* The group decipheres the two sides so that they get the same shape. Suppose the group answers are as follows:

$$\frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} = \tan \alpha - \cot \alpha$$

$$1 - 2 \cos^2 \alpha = (\tan \alpha - \cot \alpha)(\sin \alpha \cos \alpha)$$

$$1 - 2 \cos^2 \alpha = \left( \frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha} \right) (\sin \alpha \cos \alpha)$$

$$1 - 2 \cos^2 \alpha = \left( \frac{\sin^2 \alpha - \cos^2 \alpha}{\sin \alpha \cos \alpha} \right) (\sin \alpha \cos \alpha)$$

$$1 - 2 \cos^2 \alpha = \sin^2 \alpha - \cos^2 \alpha$$

$$1 - 2 \cos^2 \alpha = 1 - \cos^2 \alpha - \cos^2 \alpha$$

$$1 - 2 \cos^2 \alpha = 1 - 2 \cos^2 \alpha$$

*After the left and right sides are deciphered together, identical values are obtained.*

*Proven.*

Or

$$\begin{aligned} \frac{1-2\cos^2\alpha}{\sin\alpha\cos\alpha} &= \tan\alpha - \operatorname{ctg}\alpha \\ \frac{\sin^2\alpha + \cos^2\alpha - 2\cos^2\alpha}{\sin\alpha\cos\alpha} &= \frac{\sin\alpha}{\cos\alpha} - \frac{\cos\alpha}{\sin\alpha} \\ \frac{\sin^2\alpha - \cos^2\alpha}{\sin\alpha\cos\alpha} &= \frac{\sin^2\alpha - \cos^2\alpha}{\sin\alpha\cos\alpha} \end{aligned}$$

After the left and right sides are deciphered together, identical values are obtained.  
Proven.

Educators appreciate the efforts that have been made by the group and ask the group to discuss it again is there another way or alternative?

*Alternative Answers 6.* The group answers the question by describing the left side until the right side is obtained.

$$\begin{aligned} \text{Left side} &= \frac{1-2\cos^2\alpha}{\sin\alpha\cos\alpha} \\ &= \frac{\sin^2\alpha + \cos^2\alpha - 2\cos^2\alpha}{\sin\alpha\cos\alpha} \\ &= \frac{\sin^2\alpha - \cos^2\alpha}{\sin\alpha\cos\alpha} \\ &= \frac{\sin^2\alpha}{\sin\alpha\cos\alpha} - \frac{\cos^2\alpha}{\sin\alpha\cos\alpha} \\ &= \frac{\sin\alpha}{\cos\alpha} - \frac{\cos\alpha}{\sin\alpha} \\ &= \tan\alpha - \cot\alpha \\ &= \text{right side} \end{aligned}$$

*The left side is the same as the right side, proven.*

Or conversely, the group answers the question by describing the right side until the left side is obtained:

$$\begin{aligned} \text{Right side} &= \tan\alpha - \cot\alpha \\ &= \frac{\sin\alpha}{\cos\alpha} - \frac{\cos\alpha}{\sin\alpha} \\ &= \frac{\sin^2\alpha}{\sin\alpha\cos\alpha} - \frac{\cos^2\alpha}{\sin\alpha\cos\alpha} \\ &= \frac{\sin^2\alpha - \cos^2\alpha}{\sin\alpha\cos\alpha} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1 - \cos^2 \alpha - \cos^2 \alpha}{\sin \alpha \cos \alpha} \\
 &= \frac{1 - 2 \cos^2 \alpha}{\sin \alpha \cos \alpha} \\
 &= \text{left side}
 \end{aligned}$$

*The right side is the same as the left side, proven.*

Educators appreciate the results of discussions that have been conducted by the group.

Educators continue to go around to assist groups in solving problems given by educators. If the time to solve the problem is less than 10 minutes, the educator informs that the time for discussion is only 10 minutes and asks the students to immediately report the results of the discussion.

After the discussion time is over, the educator informs that the time for the discussion has expired, then the educator appoints four groups to advance to write and present the results of the discussion to other friends. Advanced group appointments based on workmanship strategies carried out by groups in proving trigonometric identities.

#### ***Phase 4: Develop and present the work***

After the discussion time has expired, the educator appoints four groups of students to advance writing and presenting the results of the discussion to other friends. Four groups were selected based on the work strategy carried out by the group in proving trigonometric identity.

The four groups selected are the groups that work by using the right triangle, outlining the two sides so that they get the same shape, describing the left side so that the right side is obtained, and the right side so that the left side is obtained.

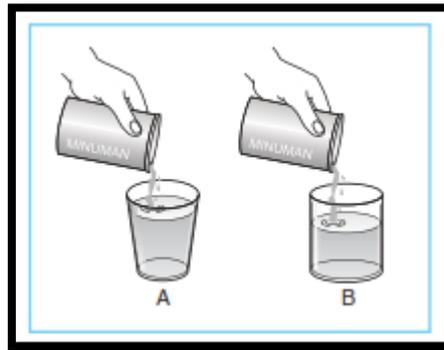
Educators along with other students monitor the four groups who write the results of their discussion on the board then the educator gives the opportunity for selected students to present their answers. After all selected students present their answers, educators give other students the opportunity to ask questions.

#### ***Phase 5: Analyze and evaluate the problem solving process***

After the four students present the group answers, the educator guides the analysis and evaluation process in accordance with the learning objectives to be achieved. Educators open the process of analysis and evaluation by asking students to review the answers and presentations of the four students. Then educators ask questions to students such as the following: What are the differences and similarities in the four answers that have been written and presented? What conclusions can be taken from the answers to the presentation results that have been displayed? How many strategies can be done to prove trigonometric identity?

Educators guide class discussion until students can find differences and proof of equality as indicated by four answers written and presented and educators guide students to conclude that there are four strategies to prove trigonometric identity, namely (1) using the aid of a right triangle, (2) decipher the left side so that the right side is obtained, (3) decipher the right side so that the left side is obtained, and (4) decipher the two sides at once so that the same shape is obtained.

Educators confirm the conclusions found by students by making illustrations or practicing directly how to prove the volume of the two glasses is different in shape but has the same volume of water. Educators provide illustrations like in the picture below:



To prove that the volume in glasses A and B is the same, it can be done by:

1. Give the water level mark to the glass B then empty the glass B and then fill the glass A into glass B, it is observed that the height of the water poured from glass A has the same height as the B cup water that is discarded. It was concluded that the contents of glass A and glass B were the same. This is like proof by describing the left side until the shape is identical to the right side.
2. In the opposite way, measuring the water level in the glass A empties the glass A and then feeds the water B into the glass A, it is observed that the height of the water poured from glass B has the same height as the discarded A water. Concluded the contents of glass B and glass A are the same. This is like proof by describing the right side until the shape is identical to the left side.
3. Take the other two measuring cups then fill the first measuring cup with the contents of the glass A and fill the second measuring cup with the contents of the glass B. Compare the contents. This is like proof by describing the left and right sides until an identical shape is obtained.

### Conclusion

Based on the discussion above, it can be concluded that the author was able to produce a learning design about the verification of Trigonometry Identity using Problem Based Learning (PBL). The flow of learning design to prove Trigonometry identity with PBL as follows: Phase 1: Student orientation on the problem. Educators convey the topic of the material and process to be carried out. Apperception. Phase 2: Organize students to learn. Educators divide students into several groups. Educators give problems. Phase 3: Guiding individual or group experience. Educators go around to provide support to groups in need. Phase 4: Develop and present the work. Educators provide opportunities for students in groups to present the results of group discussions. Phase 5: Analyze and evaluate the problem solving process. Educators guide students to draw conclusions on the learning process. Educators confirm the conclusions made by students.

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