

Design-based research in education: Getting it published

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DESIGN RESEARCH

Design research is an emerging paradigm which aims to **develop a sequence of activities** and to **grasp an empirically grounded understanding of how learning works**. (Brown, 1992; Collins, 1992; Research Advisory Committee, 1996; Gravemeijer, 1994; Cobb, Stephan, McClain, & Gravemeijer, 2001)



DESIGN RESEARCH

Design research in education involves **engineering** particular forms of learning in a natural environment such as classroom and **systematically studying** how that learning takes place in iterative cycles of learning.

(Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Collins, Joseph, & Bielaczyc, 2004; Kelly, 2003; d Lamberg & Middleton, 2009)



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Reflexive relation between theory and experiments

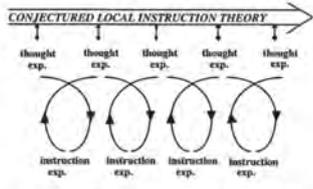


FIGURE 2 Reflexive relation between theory and experiment

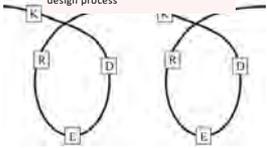
Gravemeijer (2004) Vol 6 no 2, p. 112



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DESIGN RESEARCH phases

KNOWLEDGE (K)
 Begin with knowledge of mathematics, students, students' anticipated solutions to guide the design process



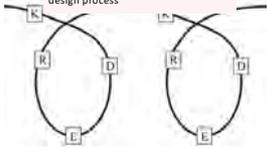
Phases of knowledge, design, experiment and retrospective analysis in *Design Research cycles* (Doik, Widjaja, Zonneveld, & Fauzan, 2010, p. 177)



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DESIGN RESEARCH PHASES

KNOWLEDGE (K)
Knowledge of mathematics, students, students' anticipated solutions, possible common misconceptions, etc.

Design (D)
Determine the mathematical goals, strategies to explore, and models to construct. Articulating hypothetical learning trajectory of students.

Experiment (E)

Phases of knowledge, design, experiment and retrospective analysis in *Design Research cycles* (Dolk, Widjaja, Zonneveld, & Fauzan, 2010, p. 177)

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DESIGN RESEARCH PHASES

KNOWLEDGE (K)
Knowledge of mathematics, students, students' anticipated solutions, possible common misconceptions, etc.

Design (D)
Determine the mathematical goals, strategies to explore, and models to construct. Articulate HLT of students.

Experiment (E)
Translate goals in the classroom activities and observe students' learning process in classroom.

Phases of knowledge, design, experiment and retrospective analysis in *Design Research cycles* (Dolk, Widjaja, Zonneveld, & Fauzan, 2010, p. 177)

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DESIGN RESEARCH PHASES

KNOWLEDGE (K)
Knowledge of mathematics, students, students' anticipated solutions, possible common misconceptions, etc.

Design (D)
Determine the mathematical goals, strategies to explore, and models to construct. Articulate HLT of students.

Experiment (E)
Translate goals in the classroom activities and observe students' learning process in classroom practice.

Retrospective analysis (R)
Analyse students' work. Reflect on experiments and an initial design to inform a subsequent design. Revisit HLT, beliefs, etc.

Phases of knowledge, design, experiment and retrospective analysis in *Design Research cycles* (Dolk, Widjaja, Zonneveld, & Fauzan, 2010, p. 177)

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Analyses in design research (1)

Analyses will have to be cases of a more general phenomenon that can inform design or teaching in other situations. One of the primary aims of a design research experiment is to support the constitution of an empirically grounded local instruction theory. (Gravemeijer & Cobb, 2013, p. 79)

Analyses in design research (2)

The objective of design experiment is not to try and demonstrate that the initial design or initial local instruction theory works. The overall goal is not even to assess whether it works... Instead the purpose of the design experiment is both to test and to improve the conjectured LIT that was developed in the preliminary phase, and to develop an understanding of how it works. (Gravemeijer & Cobb, 2013, p. 81)

Reeves (2006) depicts the design research approach as follows:

Figure 2: Refinement of problems, solutions, methods, and design principles (Reeves, 2000, 2006)

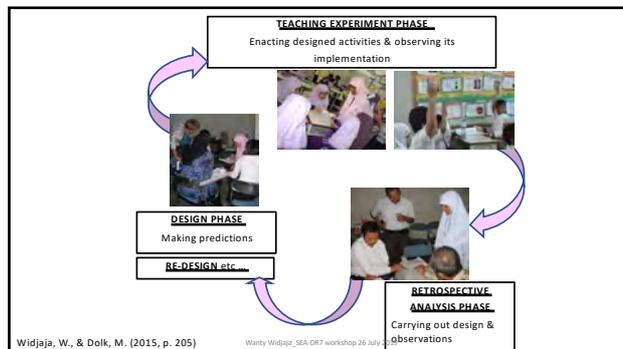
van den Akker, Bannan, Kelly, Nieveen & Plomp, T. (2013, p. 8, p.18).

Teacher DESIGN RESEARCH



Teacher design research (TDR), whose goal is to promote the **growth of teachers as adaptive experts...** the instructional aspects of TDR comes not from outside experts, but, rather from the teachers' **cognitive dissonance experiences** as designers in design cycles.

(Bannan-Ritland, 2008, p. 247)

Why design research?

- Developing appropriate activities that support learning to take place
- Examining the complexity of classroom learning practices
- Gathering a *systematic evidence* from classroom practice by research
- Encouraging researchers-teachers collaboration
- Developing a new instruction or learning theory

van den Akker, Bannan, Kelly, Nieveen & Plomp, T. (2013, p. 8, p.11)



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Theoretical intent in design research

A primary aim when conducting a retrospective analysis is to place the design experiment in a broader theoretical context, thereby framing it as a paradigm case of the more encompassing phenomena specified at the outset.

(Cobb, Confrey, diSessa, Lehrer & Schauble 2003, p. 13)



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The power of context in DBR

The strength of design studies lie in *testing the theories in crucible of practice; in working collegially with practitioners, co-constructing knowledge; in confronting everyday classroom, school, and community problems that influence teaching and learning, and adapting instruction to these conditions; in recognizing the limits of theory; and in capturing the specifics of practice and the potential advantages of iteratively adopting and sharpening theory in its context.*

(d. Lamberg & Middleton, 2009, p. 233)



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A Local Instruction Theory

Encompasses both the overall process of learning and the instructional activities that are designed to foster the mental activities that constitute the long-term process. So... a process of conjecturing and revising can happen at two levels, on the level of individual classroom sessions, and on the level of the instructional sequence as a whole.

(Gravemeijer & Cobb, 2013, p. 85)



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...On replicability and validity

...feedback from teachers on how the instructional sequence was adjusted to accommodate various classrooms can strengthen the ecological validity significantly.
(Gravemeijer & Cobb, 2013, p. 103)



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Design research to design and develop an intervention

Such as programs:

- teaching-learning strategies and materials, products and systems) as a solution to a complex educational problem as well
- as to advance our knowledge about the characteristics of these
- interventions and the processes to design and develop them, or
- alternatively to design and develop educational interventions
- (about for example, learning processes, learning



Contributions of Design research to theory and practice

Theory

- teaching-learning strategies and materials, products and systems) as a solution to a complex educational problem as well
- as to advance our knowledge about the characteristics of these
- interventions and the processes to design and develop them, or
- alternatively to design and develop educational interventions
- (about for example, learning processes, learning environments



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Methodological potential

- Multitiered, multidirectional learning by researchers, teachers and students (Bannan-Ritland, 2008; Lesh & Kelly, 2000)
- Provokes articulation and reconsideration of beliefs about teaching practice (Bannan-Ritland, 2008, p. 259)
- Strengthens collaborations between practitioners and researchers



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The strength of DESIGN RESEARCH

The strength of design studies lie in *the testing the theories in the crucible of practice; in working collegially with practitioners, co-constructing knowledge; in confronting everyday classroom, school, and community problems that influence teaching and learning and adapting instruction to these conditions; in recognizing the limits of theory; and in capturing the specifics of practice and the potential advantages from iteratively adopting and sharpening theory in its context.*

(Shavelson, Phillips, Towne, & Feuer, 2003, p. 25)



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Methodological issues and challenges

- Collecting large amount of data creates an issue of data reduction
- Tracking changes in classroom practice
- Establishing classroom mathematical norms
- Ensuring credible and trustworthy assertions



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Questions?



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My journey in using design research and getting the research published



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Theoretical underpinning

- Lesson Study
- Realistic Mathematics Education (Pendidikan Matematika Realistik Indonesia)

Design-based research methodolog




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Teacher-researcher collaboration in forming communities of inquiry



Acknowledgement: *The Implementing structured problem-solving mathematics lessons through lesson study project* by Susie Groves, Brian Doig, Colleen Vale and Wanty Widjaja



Teachers' and researchers' roles

- Teachers and coaches are taken a full authority to plan the research lesson
- The roles of Deakin research team:
 - sourcing potential mathematical tasks
 - modelling a problem solving lesson
 - providing resources e.g., research articles on Lesson Study, samples of lesson plans





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Lesson plans

- Offer insights for observers into the planning team's knowledge of curriculum, of mathematics, of teaching resources and of students' mathematical development.
- To be followed closely by the teacher during the public research lesson.

- Lesson study research theme
- About the curriculum
- Unit goals and research lesson goals
- Sequence of lessons in the unit
- Documents students anticipated solutions, strategies and reasoning.
- Flow of the lesson.
- Evaluation



Super Snooker



Super Snooker is a new snooker game that can be played at different levels with different numbers of red balls

Level 1



Level 2



Level 3



Children can play at Level 1, while experienced people can play at higher levels

I want to set up a snooker parlour for Super Snooker and am wondering how many red balls I will need to buy for Level 7

Can you help me to work this out?
Can you explain your answer with a diagram?

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Super snooker

Rationale of the Unit
Fit in Problem solving proficiency strand in the Australian Curriculum in mathematics
Jump-in lessons (not as curriculum dependent)

Objective of the lesson

- Find *multiple solutions* to the Super Snooker problem
- Use *diagram* to justify their mathematical sentences.
- See the *relationship between different solutions*.
- Understand the importance of being able to *link diagrams to the mathematical sentences*.

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Engaging in Problem solving

PROFESSIONAL LEARNING DAY, JUNE 2012



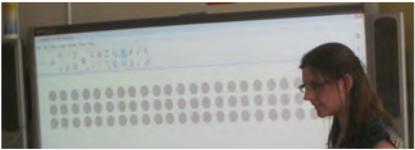


Sharing and justifying multiple strategies

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The array problem



- Here are some dots on a page. Use the diagram to show your thinking to work out how many dots there are altogether.

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Sharing your ideas

- Determine the goals and the year level
- Anticipate possible strategies using diagrams how students will solve the problem
- Identify relevant resources/teaching materials

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Engaging in Problem Solving

Teachers start with varying views and practice on problem solving

Planning starts by solving the mathematical problem




"If teachers are to encourage mathematical thinking in students, they need to engage in mathematical thinking throughout the lesson themselves." (Stacey, 2006)

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Collaborative planning



it would be easy to knowing the strategies beforehand and the progression because ...so if you got the strategies set out of the natural progression that's going to make whoever's job that is teaching a whole lot easier when they need to share.
 [Lynn, Planning meeting 3 Bobbies team]

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Anticipating students' solutions

| | | | | | | | |
|-----------------------|-------------------|--------------------------------------------------------------|------------------------|-------------------------------------------------------------|---------------------------------------------|------------------------------------------------|--------------------------------|
| Possible Solutions: | | D. Skip Counting: 3, 6, 9, 12...69 | | E. 3, 6, 9, 12...69 (Double 23) +23 | | F. 3, 6, 9, 12...69 Take 23 columns of 3 | |
| A. Don't make a start | B. Count all dots | C. Repeated Addition: 23 + 23 + 23 = 69 3 + 3 + 3...69 | G. 34 groups of 2 (+1) | | H. Groups of 6 (11 + 6) + Group of 3 (1) | | I. 23 + 3 = 69 (20 + 3 + 3) |
| | | | | J. Vertical Multiplication, Knowing that 3x3=9 and 20x3=60. | | | |



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Public research lessons

- A proving ground for planning team and teachers to test their ideas
- Not aiming to produce a perfect lesson
- Insights into students' mathematical strategies
- Insights into teachers' pedagogical decisions
- Reflective practice



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Recording students' ideas

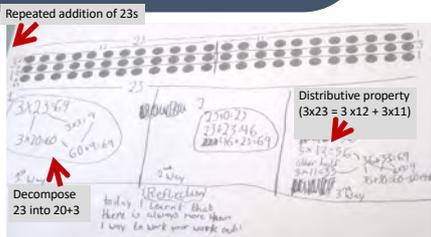


To document evidence of students' learning
 To facilitate the whole-class discussion by
pulling different ideas together

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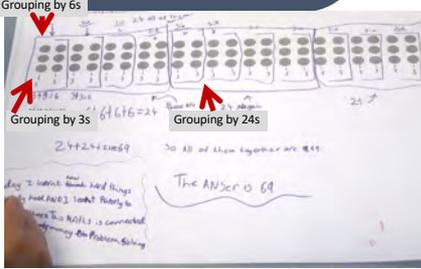
Students' multiple strategies



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Students' multiple strategies



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Role of observers



"One hundred pairs of eyes"

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Teachers' perspective

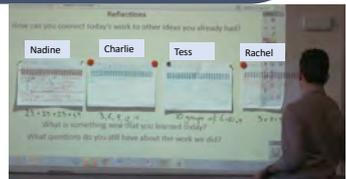
- Opportunity to develop teacher knowledge
 - Sharing of resources
 - Unpacking student thinking and misconceptions (anticipated responses)
 - Developing content knowledge: "Teach yourself before we teach the students"
- Opportunity for Reflection
 - Giving and receiving effective feedback
- Challenging each other as professionals
 - Different perspective on teaching and learning

Source: Lesson Study presentation with teachers -MAV 2012 ppt

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Planning whole-class discussion



Sequencing of recording and explanation of responses **correspond with levels of generalisation** that was indicated **in the planning**

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Post-lesson discussion



Starts with the teacher and the planning team sharing their reflection of their learning
Focuses on students' learning and evidence of their works
Gains insights from observers and the knowledgeable other

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Shared ownership of the lesson

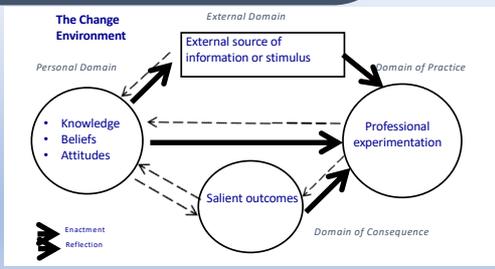
You know if you said well that it went really well or if you said that if this could have done better or that could have done better ... You've done it together, it's all yours – it's not Trevor's lesson. It's our lesson that Trevor presented.

[Camilla, planning meeting 5, 18/10/2012]

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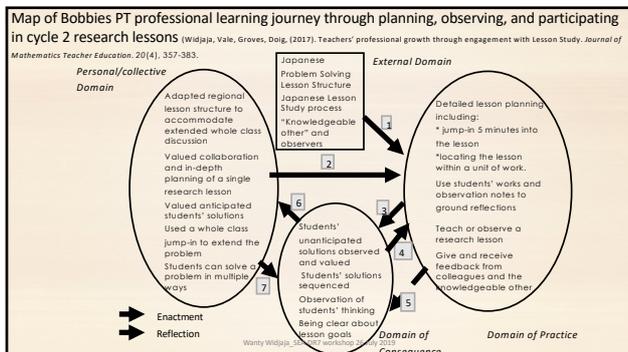


Clarke & Hollingsworth's (2002) Interconnected Model of Professional Growth (p. 951)



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Communities of inquiry

- Expectation that researchers would lead the way → ownership of the planning teams
- Collective effort by everyone in the planning teams
- A strong sense of mutual trust
- Value benefits of in-depth planning
- Planning teams working as communities of inquiry (Groves, Doig, & Splitter, 2000; Jaworski, 2008)

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Getting your work published

- By invitations in a Special issue (ZDM) or books
- Planning ahead which parts of the study fits best and you can tell a compelling narrative that is different than the others
- Meeting the timelines set by the editors – very critical if you want to get another invitation
- Present findings from your publications to increase their impact and citations

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Publish in a journal with 'the right fit' preferably a high quality journal (Scimago Q1 or Q2)

WANTY WIDJAJA, COLLEEN VALE, BRIAN DOIG

NOTHING LIKE PLANNING AND REFLECTING TOGETHER TO BUILD TRUST

Studies on issues of practicing mathematics teachers' and coaches' collaboration

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Invited book chapter arising from 'Implementing Structured Problem-solving in mathematics' project

Theorizing Professional Learning through Lesson Study Using the Interconnected Model of Professional Growth

Wanty Widjaja, Colleen Vale, Brian Doig, and Bruce Dick

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Using a conference paper as a starting point for a chapter or a journal article

To be submitted by 31 July 2019 <http://icmstudy25.ie.uilshoa.nl/discussion-document/>

ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
Theme C
Lisbon, Portugal, 3-7 February 2020

NEGOTIATING DUAL ROLES IN TEACHER COLLABORATION THROUGH LESSON STUDY: LEAD TEACHERS' PERSPECTIVE

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Lead teachers play a vital role in supporting a successful teacher collaboration. Yet, relatively little is understood about the way in which leading teachers negotiate their roles and tensions or challenges they faced to support teacher collaboration. This paper seeks to examine the dual role of lead teachers as a nursery coach or curriculum coordinator and a member of a planning team and explore the extent to which the diversity of the different teachers' experience and knowledge contribute to teacher collaboration in the context of Lesson Study. The analysis is based on interviews carried out after the first and the second Lesson Study Research cycles.

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The teaching experiment

James' revision of the second model

$\sqrt{x^2 + y^2}$
 $x > 1 + y$

- Made more realistic range in the dimensions of the step-space based on shoe-size. Measured his shoe size and found it to be 30.24 cm.
- Provided a more realistic range for the 'step height' "I feel it has to be between 15 to 23 cm maximum I think to be comfortable", "20, I think still okay... 20 you lift up a foot is not too bad. If you elevate it further, it will be difficult".

Source: Chan, Ng, Widjaja, & Seto (2015, pp. 16, 18)

The teaching experiment

James' revision of the second model

"So I only need 9 steps, 9 steps spaces"
 "Because the vertical steps is always one more than the horizontal steps because the height is always more than. So vertical steps is always one plus the horizontal".
 "Yah, 27.6666666666. So 26 and 3 thirds"

Source: Chan, Ng, Widjaja, & Seto (2015, p. 18)

Interaction between teachers and researchers

| | |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Tier 3 - Researchers | Asks questions to enable the teacher-modeller to think aloud, clarify thoughts and think more deeply during the Guide-and-Support Modelling (GSM) session. |
| Tier 2 - Teachers | Exposure to the real-world problem Formulates a mathematical solution Tests the solution Revises the solution Validates the solution as the "ideal" solution Offers it as a solution to the real-world problem |

Source: Chan, Ng, Widjaja, & Seto (2015, p. 24)

Express-test-revise cycle from a macro perspective of mathematical modelling

| | |
|----------------------|--------------------------------------------------------------------------------------------------------------------|
| Tier 3 - Researchers | Review of the modified model-eliciting task Feedback on the modified model-eliciting task |
| Tier 2 - Teachers | Review of model-eliciting task Modification of model-eliciting task |
| Tier 1 - Students | Implementation of model-eliciting task with one class Implementation of model-eliciting task with another class |

Source: Chan, Ng, Widjaja, & Seto (2015, p. 25)

The Retrospective analysis: Teachers' perception of mathematical modelling

James: I think it's like a real world situation in that sense. An authentic situation where actually the pupils need to come out with reasons and assumptions to actually solve that task. So it's a problem. And then the children need to actually use their prior knowledge to actually solve the problem. Yah and then explain the problem with of course the right reasoning adequately.

The Retrospective analysis: Researchers' reflection

- Evoke the guiding role of the teacher in designing and facilitating mathematical modelling with his students
- Aware of the level of explicitness of the guidance a facilitator
- Design questions to invoke mathematical inquiry for model development
- Extend and capture dynamic interactions between the researcher and the teacher between Tier 2-3 to be tested in Tier 1



Chan, C. M. E., Widjaja, W., & Ng, K. E. D. (2011, p. 129)

Data are collected as part of the research project by Ng, K. E. D., Chan, C. M. E., Widjaja, W. (2011). *Building Teachers' Capacity in Primary Mathematics School-Based Assessment*. Non-funded research. Singapore: National Institute of Education.




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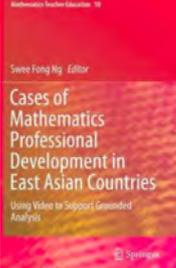
Publications arising from this research

Chan, C. M. E., Ng, K. D., Widjaja, W., & Seto, C. (2015). A case study on developing a teacher's capacity in mathematical modelling. *The Mathematics Educator* 16(1), 1-31.

Ng, K. E. D., Widjaja, W., Chan, C. M. E., & Seto, C. (2015). Activating teacher critical moments through videos for facilitating student mathematization processes during modelling tasks. In S. F. Ng (Ed.), *Cases of mathematics professional development in East Asian countries, Mathematics Teacher Education*. (pp. 15-38). Singapore: Springer

Ng, K. E. D., Chan, C. M. E., Widjaja, W., & Seto, C. (2013). Fostering teacher competencies in incorporating mathematical modelling in Singapore primary mathematics classrooms. In M. Inprasitha (Ed.), *The 6th East Asia Regional Conference on Mathematics Education (EARCOME 6): Innovations and Exemplary Practices in Mathematics Education* (Vol. 3, pp. 219-228). Phuket, Thailand: EARCOME

Ng, K. E. D., Widjaja, W., Chan, C. M. E., & Seto, C. (2012). Activating teacher critical moments of learning through reflection. *Proceedings of the 12th International Congress on Mathematical Education Topic Study Group 17* (pp. 3347-3356). Seoul: ICME.




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Assessing reasoning

Funded by Australian Academy of Science through reSolve Mathematics by Inquiry (\$122K)

Mathematical Reasoning Research Group (MaRRG)
 Assoc. Prof. Colleen Vale
 Dr. Sandra Herbert
 Dr. Leicha Bragg
 Dr. Esther Loong
 Dr. Wanty Widjaja



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Background

- Teachers' capacity to notice and use students' reasoning
- A suite of reasoning assessment materials

Aim

- Greater insights into their students' reasoning and its relationship to mathematical learning

The materials will be used by teachers as part of their everyday planning, teaching and assessment practices




Painted cube lesson




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Introducing the task to students



Allow time for students to explore and notice the pattern using concrete materials
Establish the expectation to communicate and justify their reasoning to one another



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Generalising and Conjecturing Prompt Cards

| | |
|---------------------------------------------|-----------------------------------------------|
| What is the pattern here? | Is that ... (pattern) always going to work? |
| What happens in general? | What is the rule? |
| Are there other examples that fit the rule? | How can you explain the rule to someone else? |

Produced by Mathematical Reasoning Research Group, Deakin University (2017) for ReSolve, AAMT



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| Reasoning Level | Anticipated Student Reasoning - what the student might say. | Action Analyzing (A) Forming Conjectures and Generalising (FCG) Justifying and Logical Argument (JL) | Additional notes | Sample teacher prompts to support and/or extend student thinking |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------|------------------------------------------------------------------|
| Not evident | They will always be painted because they are dipped in paint. | <ul style="list-style-type: none"> Attempts to find mathematical aspects of this examples or cases. (A) Does not communicate a common property or rule for patterns. (FCG) | | Offer MAH and/or Mins-link cubes |
| Beginning | For 1 by 1 by 1 cube there is 1 cube altogether. For 2 by 2 by 2 cube, there are 8 cubes altogether. For 3 by 3 by 3, there are 27 cubes altogether. For 4 by 4 by 4, there are 64 cubes altogether. | <ul style="list-style-type: none"> Notifies similarities across examples. (A) Recalls random known facts related to the examples. (A) | Describing what they see | Where are the cubes with no faces painted? |
| Beginning | A 1x1x1 cube will have all 6 sides painted because you dip the whole cube in the paint | <ul style="list-style-type: none"> Describing what they did and why it may or may not be correct. (JL) Recognises what is correct or incorrect using materials, objects, or etc. | Describing not analysing - what the student can "see" | Where are the cubes with 1, 2, 3 faces painted? |



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| | | | | |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------|--|
| Developing | To find the total number of small cubes in $n \times n \times n$. For example, in a 3 side cube, you just times 3 by 3 by 3 to get the total number of cubes. | <ul style="list-style-type: none"> Communicates a rule about a property using words, diagrams or number sentences. (FCG) Checks the truth of statements using materials and informal methods. (JL) Uses known facts to verify that the statement, common property, or rule for a pattern holds for each case. (JL) | Identifying a "rule" for calculating volume. | |
| Consolidating | 1-side painted: Inside a 5x5x5 is actually a 3x3x3 cube, so you only count those cubes (3+3+3) which will be painted on 1 side. So if it's a 6-sided cube, you have to take away 2, which will make it a 4x4x4 on the inside, therefore only 16 | <ul style="list-style-type: none"> Repeats and extends patterns using both the numerical and spatial structure. (A) Notifies more than one common property. (A) | | |

DMAT produced by Mathematical Reasoning Research Group, Deakin University (2017) for ReSolve, AAMT



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Cube

Imagine a cube made up of 27 smaller cubes (3 x 3 x 3). Imagine that you dip the cube in paint.

If you now separate it into 27 small cubes, you will notice that some of the small cubes are painted. Which small cubes have been painted and how many faces are painted? (1 face, 2 faces, or 3 faces, but not painted at all - not visible, keep one face?)

Fill in the grid below for a 3 x 3 x 3 cube.

| Size | Total number of small cubes | Number of cubes painted on 3 faces | Number of cubes painted on 2 faces | Number of cubes painted on 1 face | Number of unpainted cubes |
|----------|-----------------------------|------------------------------------|------------------------------------|-----------------------------------|---------------------------|
| 1x1x1 | 1 | 0 | 0 | 0 | 1 |
| 2x2x2 | 8 | 0 | 0 | 0 | 8 |
| 3x3x3 | 27 | 0 | 12 | 6 | 9 |
| 4x4x4 | 64 | 0 | 24 | 24 | 16 |
| 5x5x5 | 125 | 0 | 36 | 54 | 35 |
| 6x6x6 | 216 | 0 | 48 | 84 | 64 |
| 7x7x7 | 343 | 0 | 60 | 112 | 101 |
| 8x8x8 | 512 | 0 | 72 | 144 | 156 |
| 9x9x9 | 729 | 0 | 84 | 180 | 213 |
| 10x10x10 | 1000 | 0 | 96 | 216 | 274 |

Describe a rule for predicting the pattern for larger cubes without counting all the small cubes! Show one working for your class!

Student Name: _____

For columns A, B, C, D, and E, describe the pattern that you see. What changes, and what stays the same?

A. _____

B. Always 8

C. _____

D. _____

E. The answer is A from 2 cubes up

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The Painted Cube

Imagine a cube made up of 27 smaller cubes (3 x 3 x 3). Imagine that you dip the cube in paint.

If you now separate it into 27 small cubes, you will notice that some of the small cubes are painted. Which small cubes have been painted and how many faces are painted? (1 face, 2 faces, or 3 faces, but not painted at all - not visible, keep one face?)

Fill in the grid below for a 3 x 3 x 3 cube.

| Size | Total number of small cubes | Number of cubes painted on 3 faces | Number of cubes painted on 2 faces | Number of cubes painted on 1 face | Number of unpainted cubes |
|----------|-----------------------------|------------------------------------|------------------------------------|-----------------------------------|---------------------------|
| 1x1x1 | 1 | 0 | 0 | 0 | 1 |
| 2x2x2 | 8 | 0 | 0 | 0 | 8 |
| 3x3x3 | 27 | 0 | 12 | 6 | 9 |
| 4x4x4 | 64 | 0 | 24 | 24 | 16 |
| 5x5x5 | 125 | 0 | 36 | 54 | 35 |
| 6x6x6 | 216 | 0 | 48 | 84 | 64 |
| 7x7x7 | 343 | 0 | 60 | 112 | 101 |
| 8x8x8 | 512 | 0 | 72 | 144 | 156 |
| 9x9x9 | 729 | 0 | 84 | 180 | 213 |
| 10x10x10 | 1000 | 0 | 96 | 216 | 274 |

Describe a rule for predicting the pattern for larger cubes without counting all the small cubes! Show one working for your class!

ANALYSING: Noticing more than one common property by systematically listing facts about number of cubes.
You do observe student constructing this table to be sure that they used a rule that they noticed rather than counting using concrete materials.

Student Name: _____

For columns A, B, C, D, and E, describe the pattern that you see. What changes, and what stays the same?

A. _____

B. It's repeating 8 because there are only 8 corners

C. It's going up by 12 every bigger shape

D. _____

E. _____

JUSTIFYING & LOGICAL ARGUMENT: Verifies truth of a statement using a common property or known facts.

FORMING CONJECTURES AND GENERALISING: Communicates the rule about a pattern using words.

FORMING CONJECTURES AND GENERALISING: Explains the meaning of the rule using one example.

ANALYSING: Considering FORMING CONJECTURES & GENERALISING & JUSTIFYING & LOGICAL ARGUMENT: Developing Teacher Prompt: Can you write a rule to find the number of cubes painted on two sides for any size cube? Can you describe the pattern for the number of cubes with one painted side?

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Communicating reasoning

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Reasoning Trajectories

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| | Analysing | Forming Conjectures and Generalising | Justifying and Logical argument |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Not evident | <ul style="list-style-type: none"> Does not notice numerical or spatial structure of examples or cases. Attempts to non-mathematical aspects of the examples or cases. | <ul style="list-style-type: none"> Does not communicate a common property or rule for pattern. Non-systematic recording of cases or patterns. Random facts about cases, relationships or patterns. | <ul style="list-style-type: none"> Does not justify. Appeals to teacher or others. |
| Beginning | <ul style="list-style-type: none"> Notifies similarities across examples Recalls random known facts related to the examples. Repeats and extends patterns displayed visually or through use of materials. Attempts to sort cases based on a common property. | <ul style="list-style-type: none"> Uses body language, drawing, counting and oral language to draw attention to and communicate: <ul style="list-style-type: none"> A single common property repeated components in patterns. Adds to patterns displayed verbally and/or visually using diagrams or through use of materials. | <ul style="list-style-type: none"> Describes what they did and why it may or may not be correct. Recognises what is correct or incorrect using materials, objects, or words. Makes judgements based on simple criteria such as known facts. The argument may not be coherent or include all steps in the reasoning process. |
| Developing | <ul style="list-style-type: none"> Notifies a common numerical or spatial property. Recalls, repeats and extends patterns using numerical structure or spatial structure. Sorts and classifies cases according to a common property. Orders cases to show what is the same or stays the same and what is different or changes. Describes the case or pattern by labelling the category or sequence. | <ul style="list-style-type: none"> Communicates a rule about a: <ul style="list-style-type: none"> property using words, diagrams or number sentences. pattern using words, diagrams to show recursion or number sentences to communicate the pattern as repeated addition. Explains the meaning of the rule using one example. | <ul style="list-style-type: none"> Verifies truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods. Refutes a claim by using a counter example. Starting statements in a logical argument are correct and accepted by the classroom. Detecting and correcting errors and inconsistencies using materials, diagrams and informal written methods. |

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| Consolidating | <ul style="list-style-type: none"> Notifies more than one common property by systematically generating further cases and/or listing and considering a range of known facts or properties. Repeats and extends patterns using both the numerical and spatial structure. Makes a prediction about other cases: <ul style="list-style-type: none"> with the same property included in the pattern. | <ul style="list-style-type: none"> Identifies the boundary or limits for the rule (generalisation) about a common property. Explains the rule for finding one term in the pattern using a number and/or sentence. Extends the number of cases or pattern using another example to explain how the rule works. Extends the generalisation using logical argument. | <ul style="list-style-type: none"> Uses a correct logical argument that has a complete chain of reasoning to it and uses words such as 'because', 'if...then...', 'therefore', 'and so', 'that leads to' ... |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Extending | <ul style="list-style-type: none"> Notifies and explores relationships between: <ul style="list-style-type: none"> common properties numerical structures of patterns. Generates examples: <ul style="list-style-type: none"> using tools, technology and modelling to form a conjecture. | <ul style="list-style-type: none"> Communicates the rule for any case using words or symbols, including algebraic symbols. Applies the rule to find further examples or cases. Generalises properties by forming a statement about the relationship between common properties. Compares different symbolic expressions used to define the same pattern. | <ul style="list-style-type: none"> Uses a watertight logical argument that is mathematically sound and leaves nothing unexplained. Verifies that the statement is true or the generalisation holds for all cases using logical argument. |

Evidence of student's reasoning

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Paint Cube

ANALYSING: Noticing more than one common property by systematically listing facts about number of cubes.

Student Name: _____

For column A, B, C, D, and E, describe the patterns that you see. What changes, and what stays the same?

JUSTIFYING & LOGICAL ARGUMENT: Verifies truth of a statement using a common property or known facts.

A: It's repeating 8 because there are only 8 units.

B: It's going up by 12 every bigger shape.

FORMING CONJECTURES AND GENERALISING: Communicates the rule about a pattern using words.

if you got for example a 3x3x3 cube you don't count the single outline of one face that leaves 1 so answer is 12x6 and you have your answer.

FORMING CONJECTURES AND GENERALISING: Explain the meaning of the rule using one example.

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Challenges and issues

- The absence of discussion of epistemological issues (Walker, 2011)
- How do we know that the effects observed are causally related to the design? How do we know they are not related to any of the other elements of the context, or what the specific combination of design and context was?

.... only when we know what makes a design work can we make suggestions regarding its applicability in other instructional settings. (Reimann, 2011, p. 43)

- Data deluge: complexities of planning and documenting the multi-cyclical process of design and theory revision. (Reimann, 2011, p. 47)

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Final remark

Educational innovations should not be a net addition to what teachers do. For teachers *innovations* have sometimes become synonymous with centre-led, top-down *initiatives*, which have indeed often been an addition to what teachers do rather than a replacement and this explains in part why some of them have been resisted and treated as a burden.

(Hargreaves, 2004, p. 66)



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