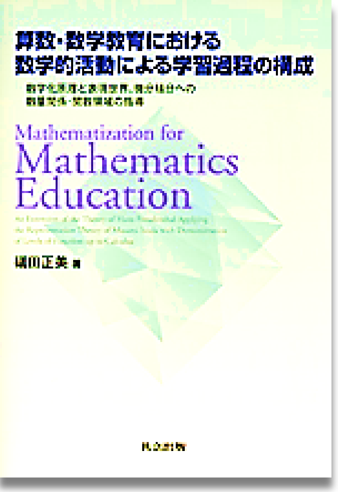
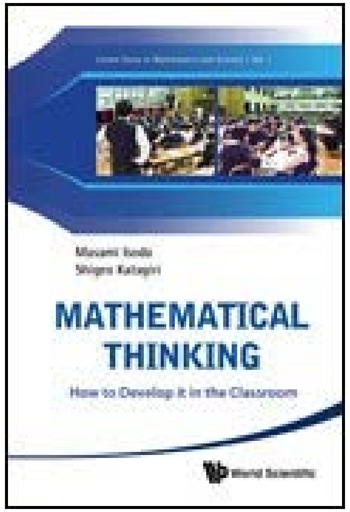
Keynote Lecture at the The 7th South East Asia Design Research International Conference

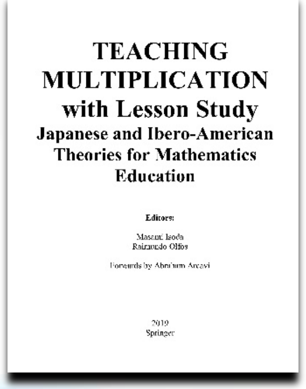
Task Sequence Theories for Curriculum Design to Develop Mathematical Thinking

Behind Theory of Indonesia Edition of Japanese Textbook





Based on: Freudenthal (1973), Isoda (1987, 1996, 2015)
Isoda & Katagiri (2012)



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What Math We Teach: ASEAN Standards

http://www.recsam.edu.my/sub_SEA-BES/index.php/ccrls

Consciousness of Value
Strangle for Existence

Nakajima, 1983
- Usefulness and Necessity
- Habit of Mind
- Culture
- **Activity for Enjoyment**

Mathematical Values, Attitudes and Habits for Human Character

Appreciation

Mathematical Values:
Generality and Expandability
Reasonableness and Harmony
Usefulness and Efficient
Simpler and Easier
Beautifulness

Mathematical Attitude attempting to:
See and think mathematically
Pose question and develop explanation such as why and when
Generalize and extend
Appreciate others' idea and change representation to conceptualize

Habits of mind for Citizen to live:
Reasonably and critically with respecting and appreciating others
Autonomously Creatively and innovatively in harmony
Judiciously using tools such as ICT
Empowerly in imagining the future through lifelong learning

Mathematical Thinking and Processes

Reflection

Mathematical Ideas for:
Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.

Mathematical Thinking:
Generalization and Specialization
Extension and Integration
Inductive, Analogical and Deductive reasoning
Abstracting, Concretizing and Embodiment
Objectifying by representing and symbolizing
Relational and Functional thinking
Thinking forward and backward

Mathematical Activities for:
Problem Solving
Exploration and Inquiry
Mathematical Modeling
Conjecturing, Justifying and Proving
Conceptualization and Proceduralization
Representation and Sharing

Content

- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures and Solids
- Pattern & Data Representations

- Extension of Number and Operations
- Measurement & Relations
- Plane Figures & Space Solids
- Data Handling & Graphs

- Number & Algebra
- Space & Geometry
- Relationship & Functions
- Statistics & Probability

Acquisition

Experience through Mathematization (Freudenthal)
Extension and Integration (Japanese Curriculum,)

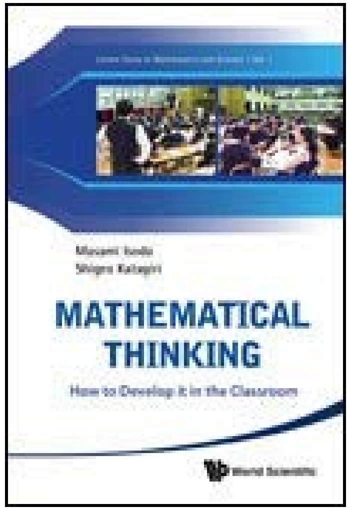
Questions

- **How do you say my process of teaching is better than you?**
  - **It is depending on objective (Isoda 2014, Mongoo, Jahan, Isoda 2017)**

Human Character Formation
Value
Attitude

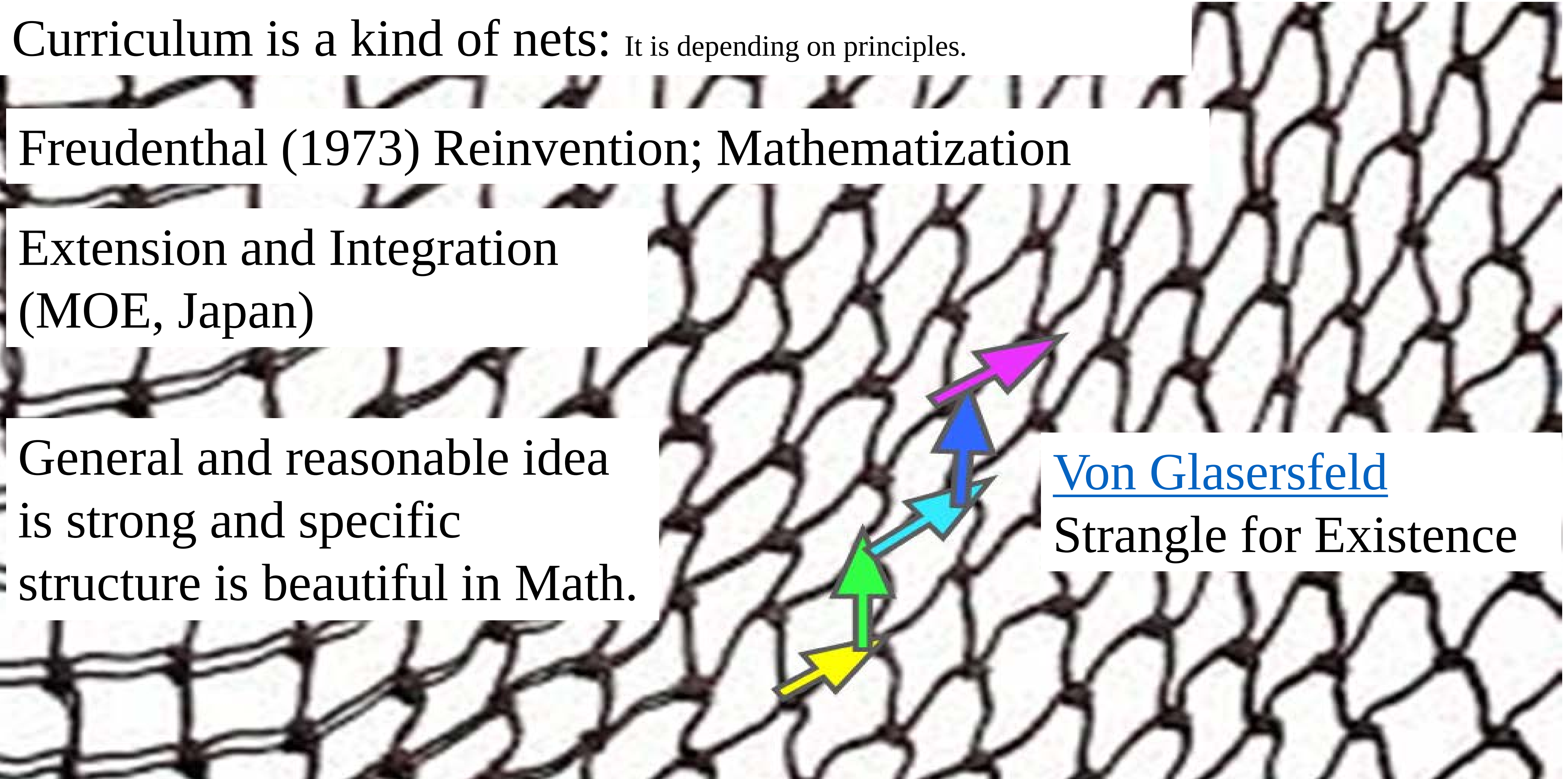
Skills: The Way to Think
Mathematical Thinking

Knowledge
Mathematical Knowledge and Skills



- **Reality is depending on students' value.**
  - **What is mathematical process?**
  - **What is mathematical activity?**
  - **What is mathematics?**
- **It might be developed through teaching and learning through curriculum sequence**
  - **What is teaching material?**
  - **Teaching Material = Content + Objective under the Curriculum Seauence**
- **Principle: Developing students who learn mathematics by and for themselve**

Task Sequence for What?



Curriculum is a kind of nets: It is depending on principles.

Freudenthal (1973) Reinvention; Mathematization

Extension and Integration (MOE, Japan)

General and reasonable idea is strong and specific structure is beautiful in Math.

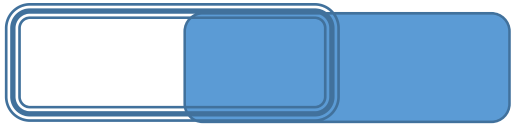
Von Glasersfeld

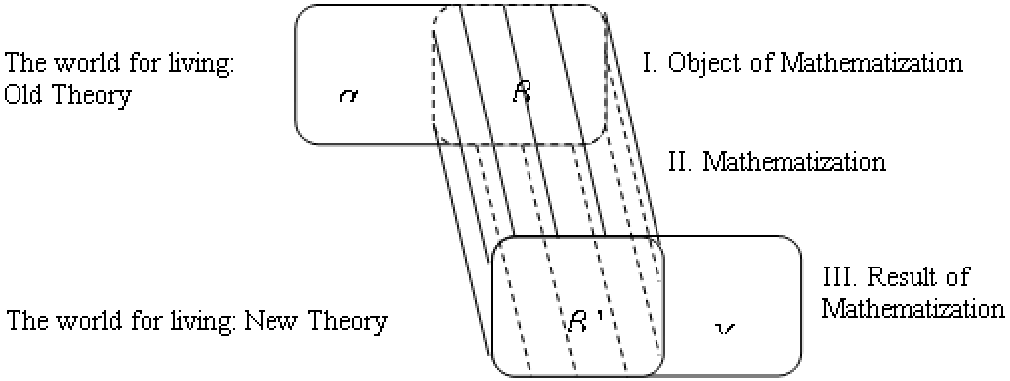
Strangle for Existence

Theories for Task Sequence by Isoda

http://iwme.jp/pdf/Proceedings_IWME2018.pdf

- Levels for Mathematizations depending on the **Organizing Principle**



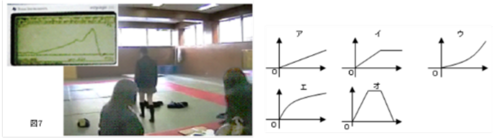


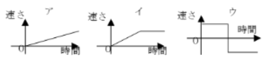
- Conceptualization and Proceduralization

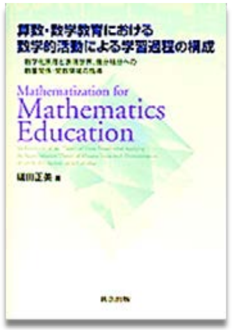
Keywords:
Contradiction and Dialectic for Conceptual Changes
Known to Unknown
Make Sense to Sense Making

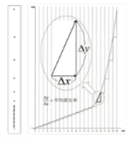
Sense Making for Make Sense to learn Math by and for themselves

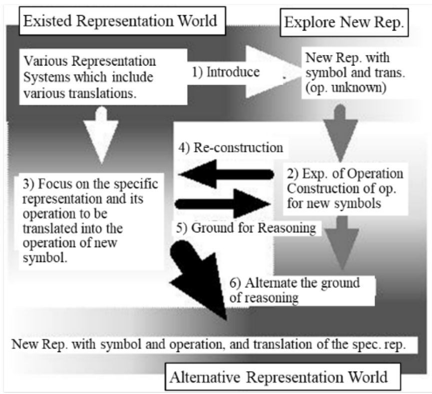
Theory for Representation

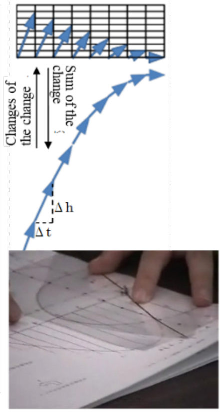


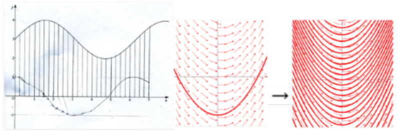


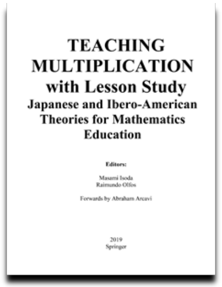


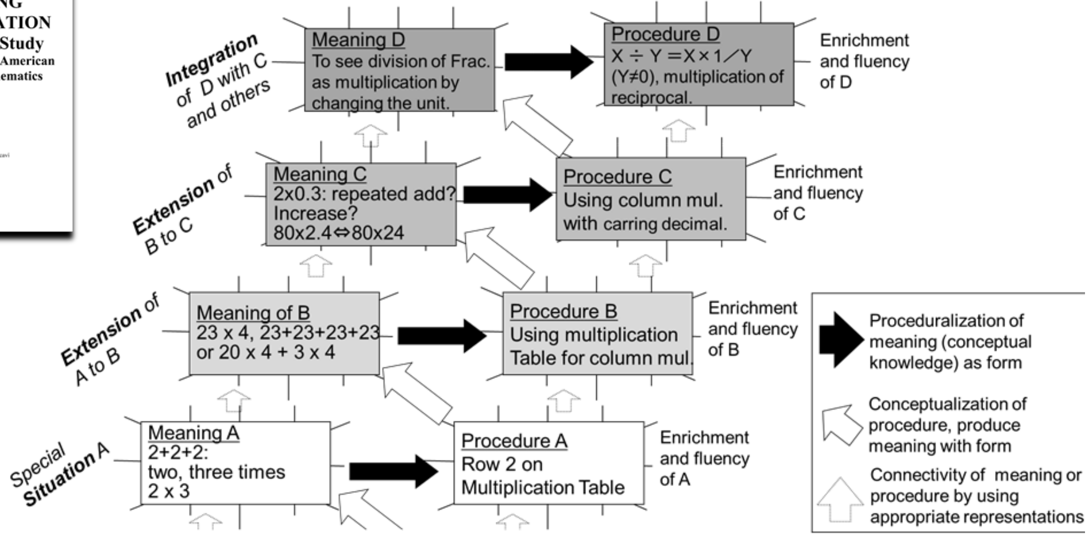








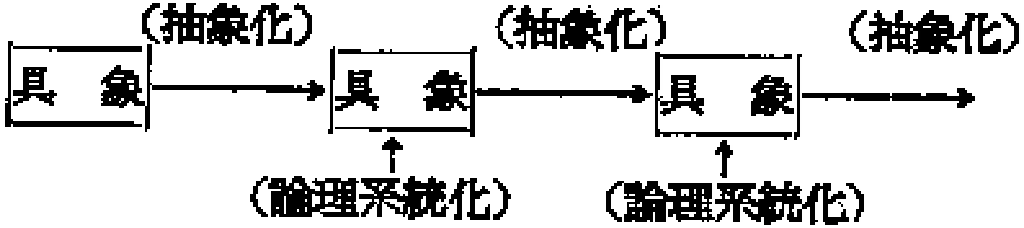




Definition of Mathematization

- Mathematization was the basic concept for the teaching sequence of the content of the middle school textbooks: (Textbook under MOE, 1943)
- Mathematization using the terminology of “[Embodiment]” and “→ (Abstraction)” with “(↑Logical Systematization)”

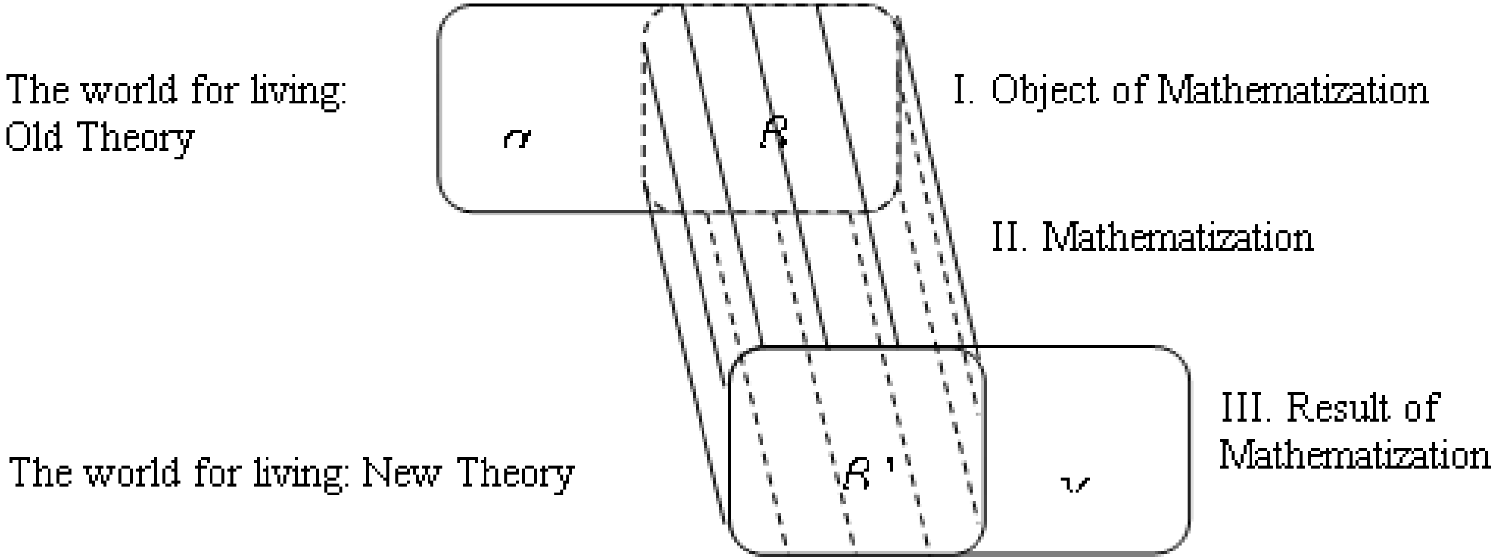
  (Nabeshima and Tokita ,1957)



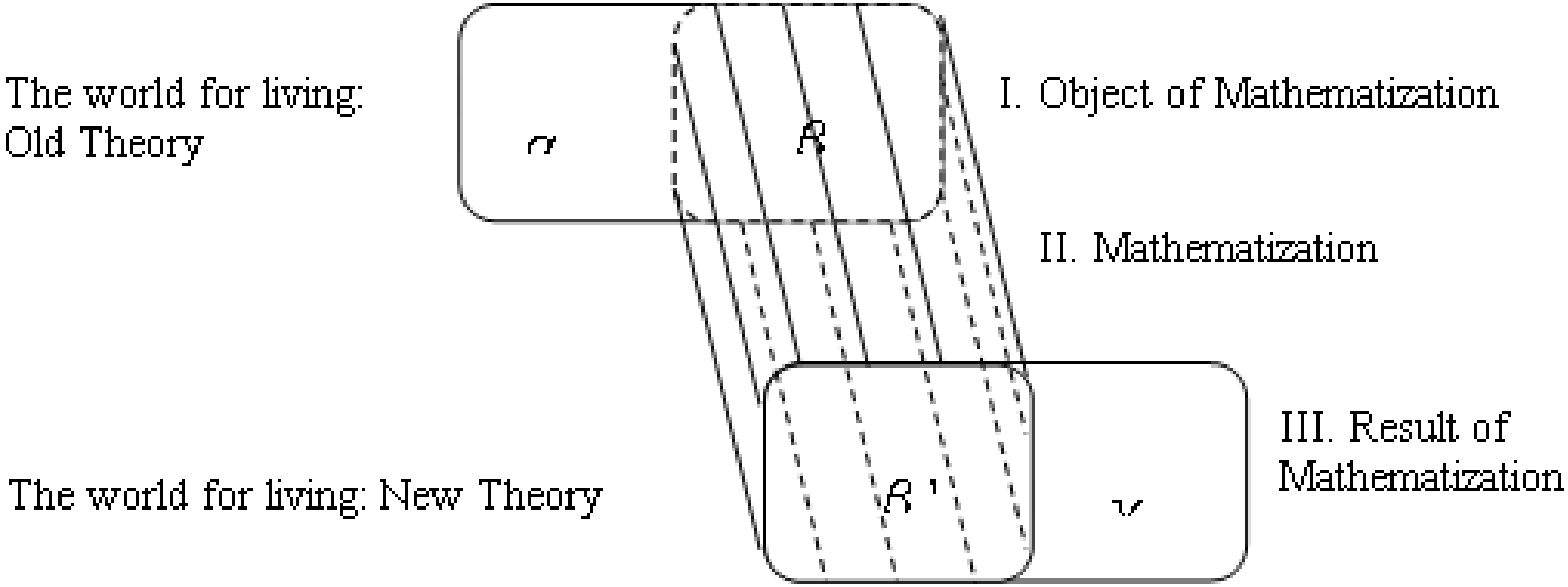
- Freudenthal defined mathematization by the re-organization of (mathematical) experiences by mathematical means (1973)

Isoda (1984, 2012, 2015) summarized Freudenthal's mathematization as follows:

1. Mathematization is the **reorganization** of experiences by using the mathematical methods.
2. The process of mathematization is described with levels:
   I. **Object** of Mathematization: Experiences are condensed through the activity of lower level mathematical methods.
   II. **Mathematization**: Methods of the lower level become the object of the upper level. Mathematical methods and experiences of the lower level are reorganized.
   III. **Result** of Mathematization: Experiences of the upper levels are condensed through the activities in that level.



Horizontal & Vertical for What?



Isoda (1984, 2012, 2015) summarized Freudenthal's mathematization as follows:

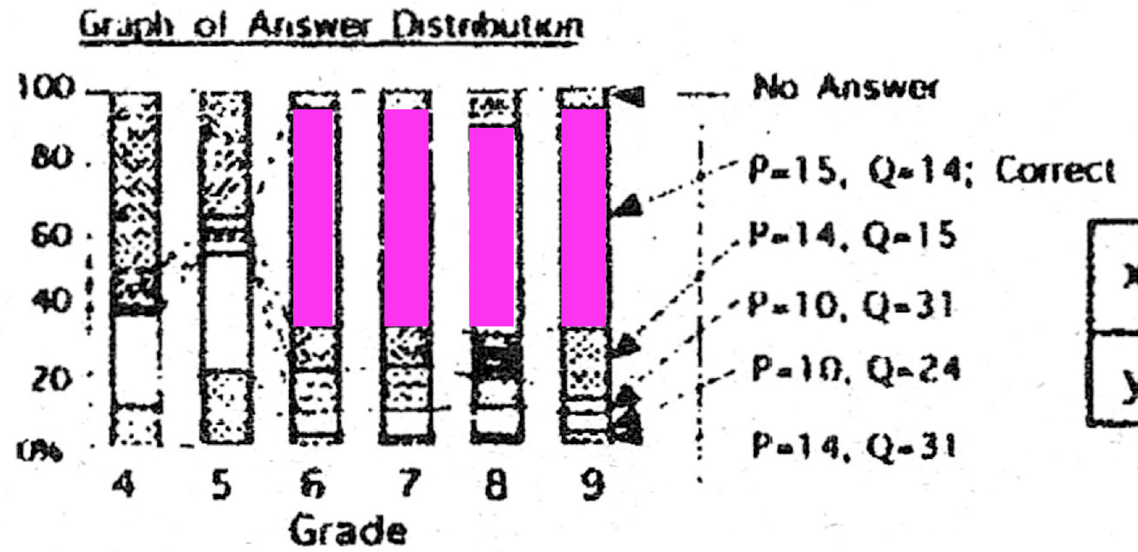
3. **Levels of Activity** for living by Freudenthal and **Levels of Thinking** by van Hiele: Both of them referred levels to explain **dis-continuity of learning process**. Levels of Activities are described by the content of activity in relation to the organizing principle. Levels of Thinking are described as the difference of systems and languages with exemplar of van Hiele Levels in Geometry. Both levels have the following features:

- Every level has its own method in mathematics.
- Levels of Activity describe the **different** mathematical **intuitions** and Levels of Thinking describe the **different languages** in mathematics.
- Discontinuity: The difference of levels emerged as the **contradiction** or the difficulty of translations without appropriate terms for explanations.
- Duality: The relationship between levels is the **methods** used for the lower level to **become the object** of the upper level.

	The Levels of Geometry	The Levels of Function
Level 1	Students explore matter (object) using ***shapes*** (method)	Students explore phenomena (object) using unsophisticated ***relations*** *or* ***variation*** (method)
Example of conflicts between levels	*Because it has rounded corners, the road sign board 'YIELD' is not a triangle according to the meanings of Level 2, but we call the shape as a triangle in daily language.*	*In Japanese, we use "2 BAI, 3BAI" to mean "two times, three times" on level 2. But in everyday Japanese (Level 1), we can use "BA1" to mean either "double" or "plus". A child on level 1 says "BAI, BAI" ("plus plus") to mean three times the original amount. But "BAI, BAI" ("double double") usually means four times. On Level 2, students use "2 BAI, 3 BAI" to explain proportion as a covariance and they say three times as "3BAI" and do not say it "BAI, BAI".*
Level 2	Students explore the figures using *the* ***properties***. The object on level 2 was the method on level 1.	Students explore the relations using ***rules***. The object on level 2 was the method on Level 1.
Ex. of conflicts	*A square is rectangular on Level 3, but not on Level 2.*	*The constant function is a function on Level 3 but 'constant' is not the relation which was discussed as covariation on Level 2.*
Level 3	Students explore the properties of figures using ***implication***.	Students explore the rules using ***notations of functions***.
Example of conflicts	*The isosceles triangle has congruent angles. On Level 3, it is induced already and we do not have to explain more. On Level 4, we prove it.*	*On Level 3, a tangent line of quadrilateral function deduce using the property of only one common point/multiple root. On the Level 4, the tangent line does not always have this property.*
Level 4	Students explore the proposition, which is formed by implication, using ***proof***.	Students explore functions using ***derived or primitive function.***

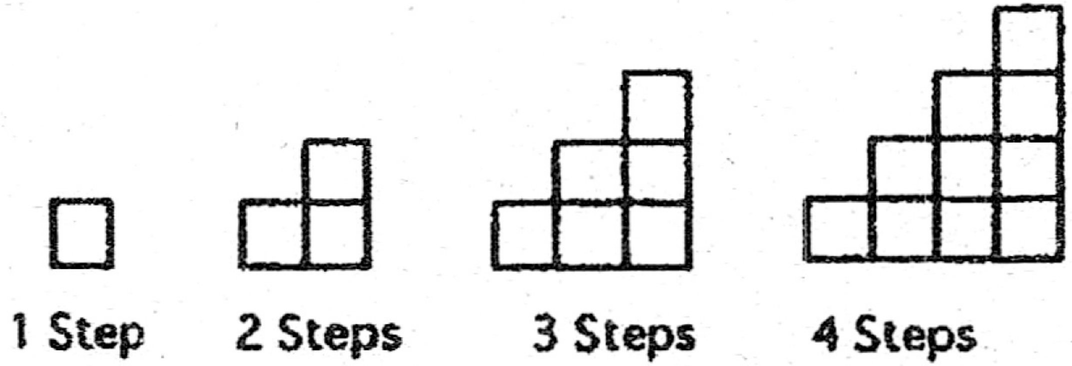
The Case of Ratio and Propotion

Problem 1 In the right table, if y is in proportion to x, then select the pair which is appropriate for P and Q in the table.



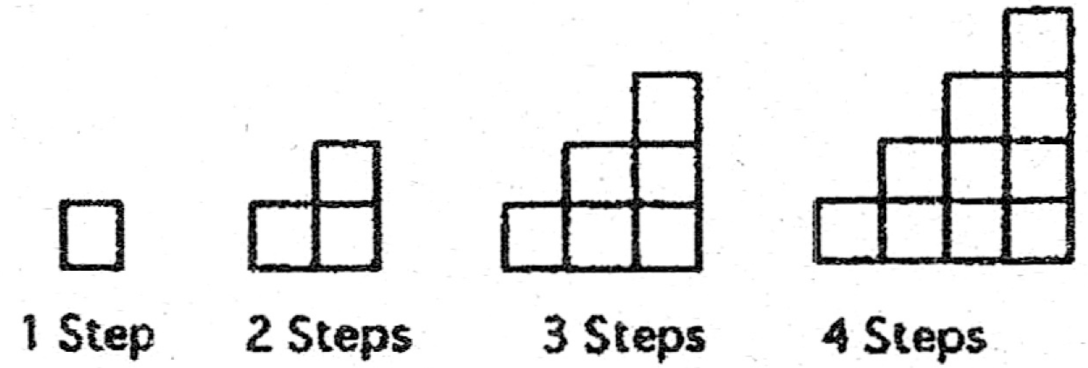
x	3	6	P
y	7	Q	35

Problem 2. Let's make stairs using squares with sides 1 cm as follows.

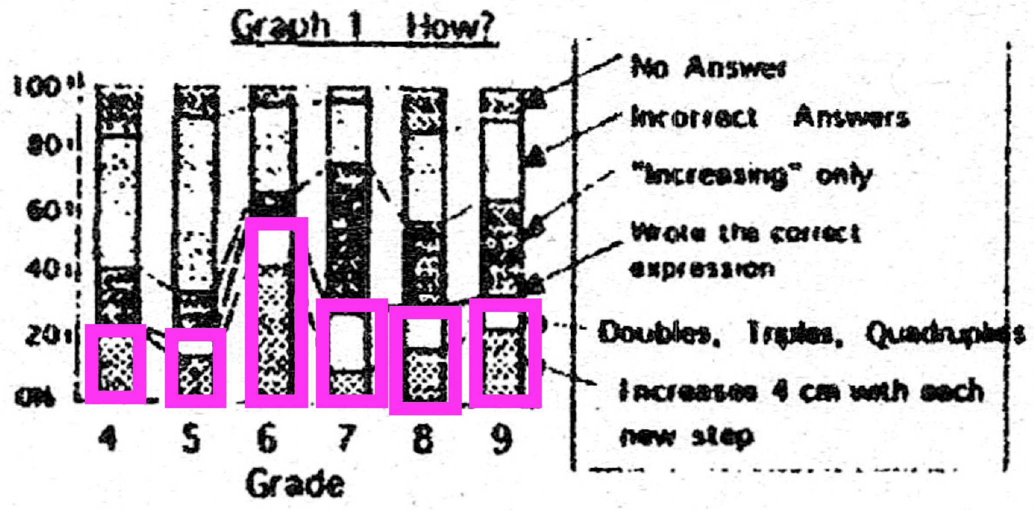


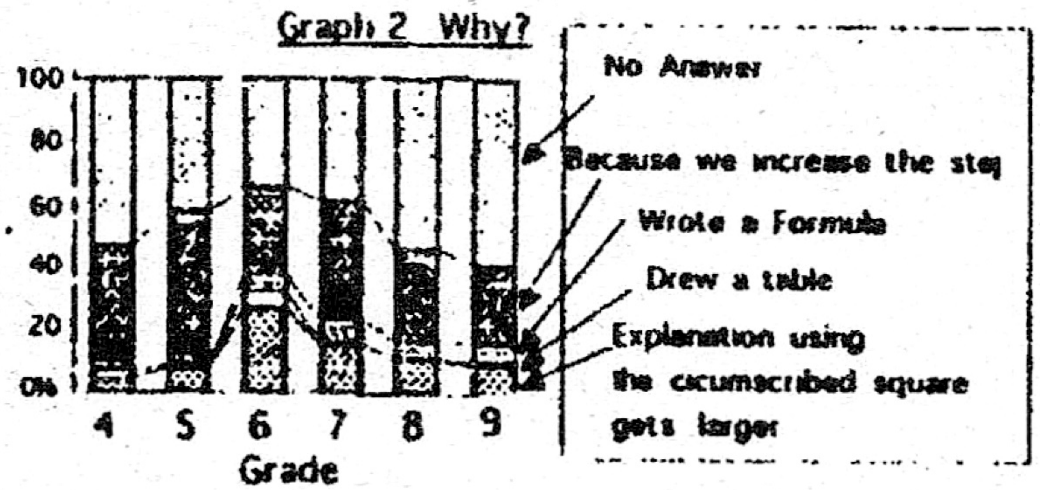
Grade

Problem 2. Let's make stairs using squares with sides 1 cm as follows.

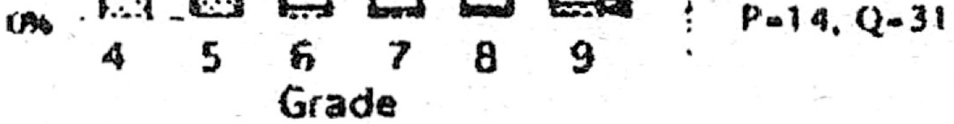


Q1. How does the perimeter change as the number of steps increases? Why do you think so?





Q2. How can we relate the number of steps and the perimeter?
Q3. What is the perimeter if there are ten steps?



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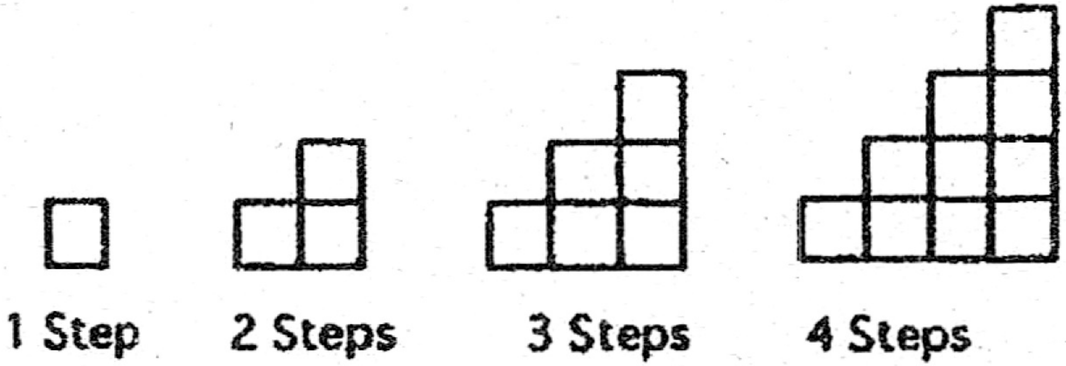

Q2. How can we relate the number of steps and the perimeter?
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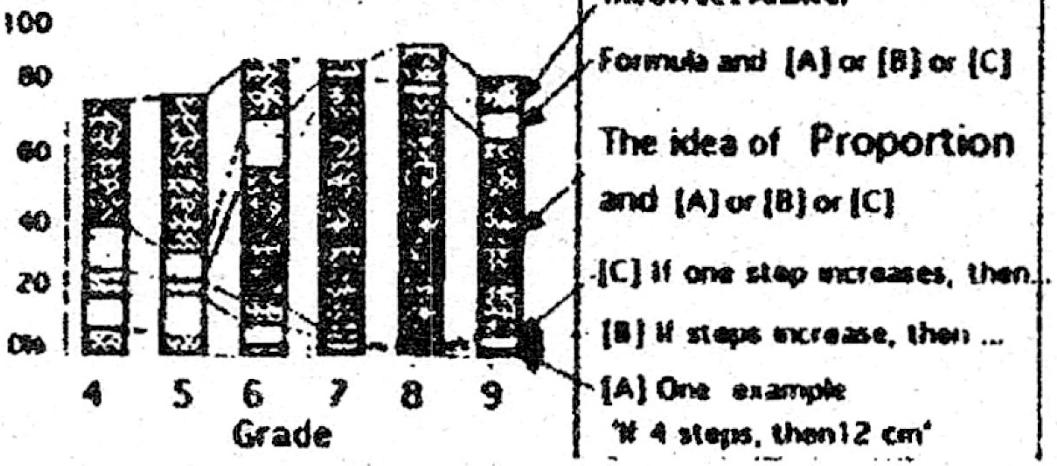
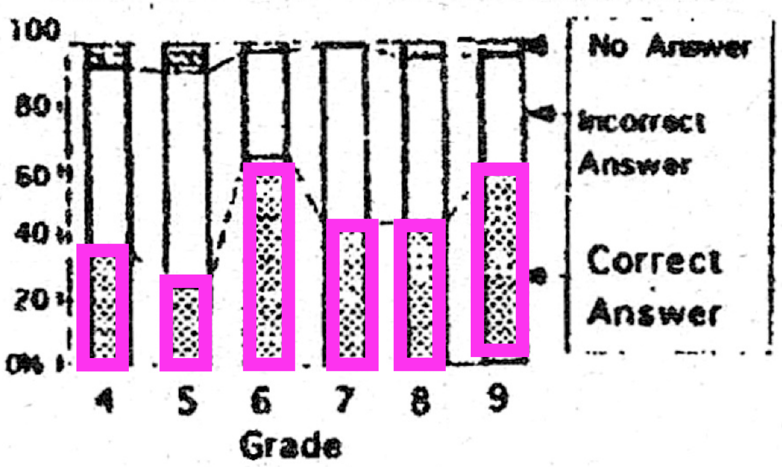
Graph 3. (Q2) Relation



Graph 4. (Q3) Application of Relation



Levels for Proportion

Level of Function	Explanation of Proportionality
Level 1	**Daily language**: It is difficult to distinguish linearity and proportionality.
Level 2	**Relations among quantities**: Proportionality is defined by the table
Level 3	**Algebra and Geometry**: Proportion is defined by expression or graphs.
Level 4	**Calculus**: Proportion is applied to the differential equation.

Masaaki Ogasawara mentioned Kawazoe's work.

- What is meaningful?

Representation Theory for Mathematization (Isoda 1991)

- Representation: A set of representation produces context / Objective.
- Method of Representation: R(Symbol ; **Operation**)

  Such as Algebraic, Geometric, Graphical Representation and so on.

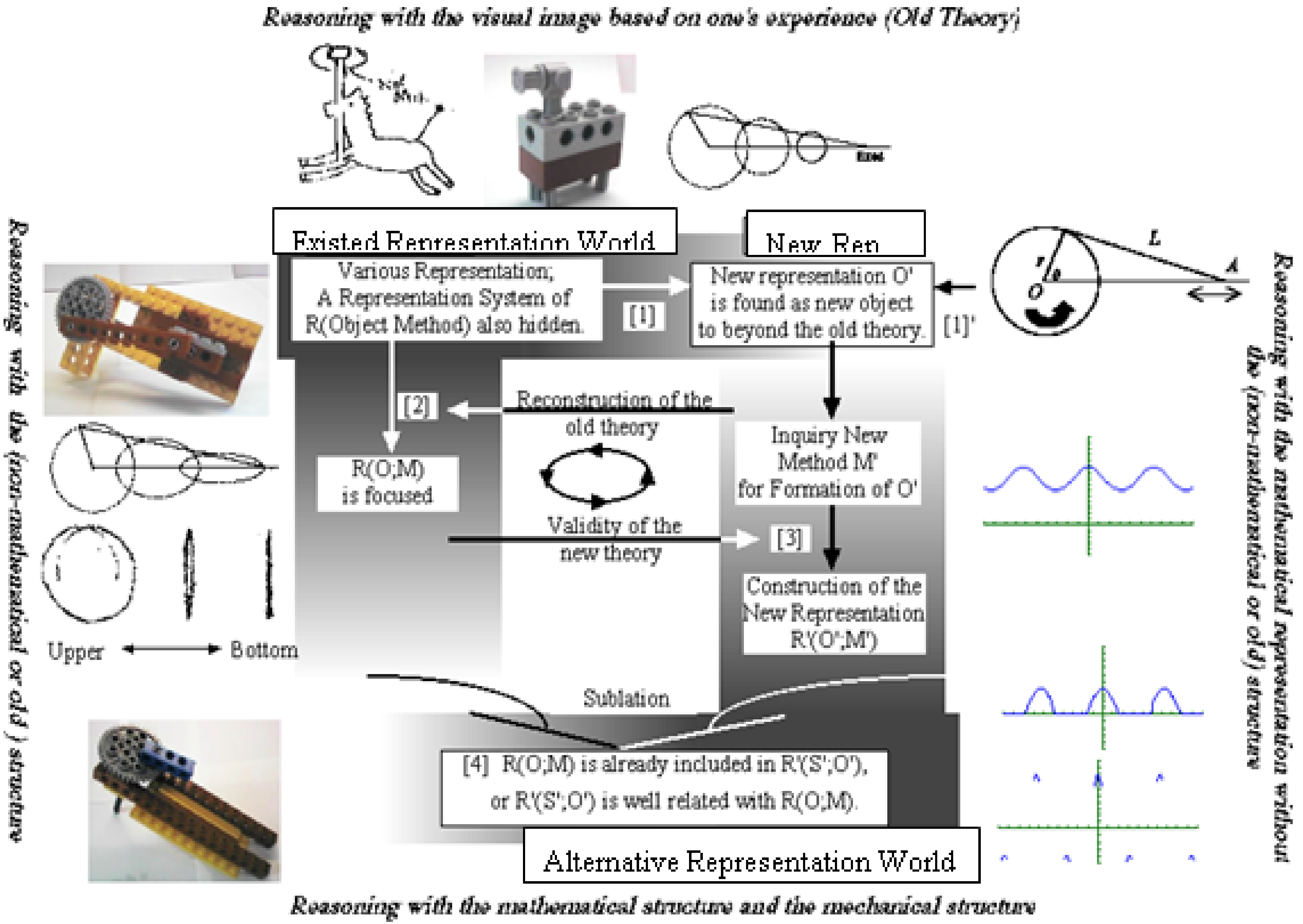
- Translation Between Representations: Translation Rule
- Representation System: A set of the Methods of Representations
- World of Representations: In relation to given tasks, we chose the representation, Methods of Representation and Representations system
- Meaning:
  - Procedural Meaning within a method of representation
  - Conceptual Meaning through *translation between different methods of representation*
- *Shift the Worlds of Representations from the one world to the others*.

$x=3$

$2(x-1)=4$

$2x=6$

An Example for the Representation Theory for Mathematization



Mechanism for **Merry-go-round**

difference

Crank Mechanism

difference

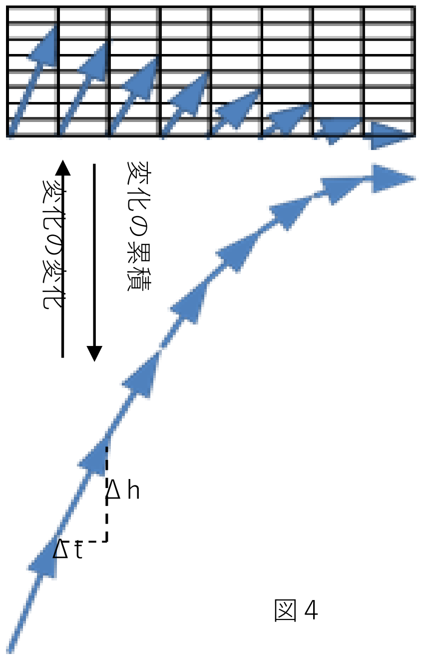
Graph of Crank Mechanism

difference

Crank Mechanism controlled by **Parameters**

Fundamental Theorem of Calculus

Level of Function	Explanation of Content with Activity for Fundamental Theorem for Calculus
Level 1	**Daily language**: On the car, the acceleration on the speed meter is felt as the pressure to our back on the seat. Fill the water into the bottle.
Level 2	**Relations among quantities**: Changes of the slopes on the line graph. Area on the graph: Speed x Times = Distance
Level 3	**Algebra and Geometry**: The rate of changes of various functions such as linear function, quadratic function and so on.
Level 4	**Calculus**: Using the fundamental theorem of calculus.



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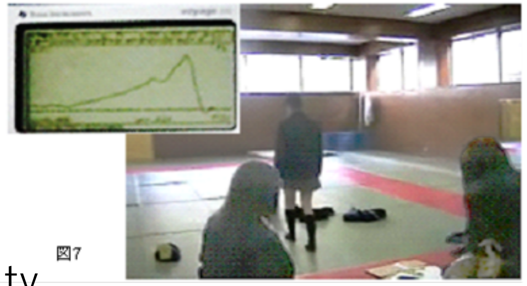
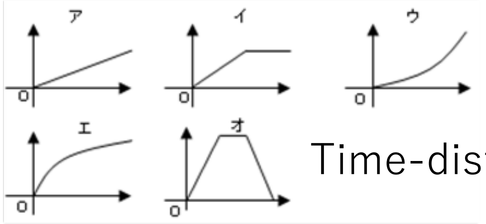
At High School Math II, Calculus is introduced without Limit.

On Math III, it is re-introduced with Limit.

What is the difference?

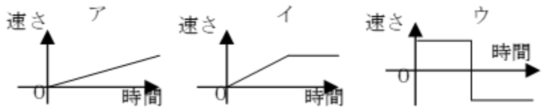
Mathematization on Fundamental Theorem of Calculus

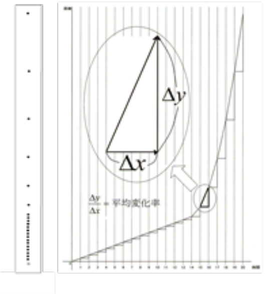


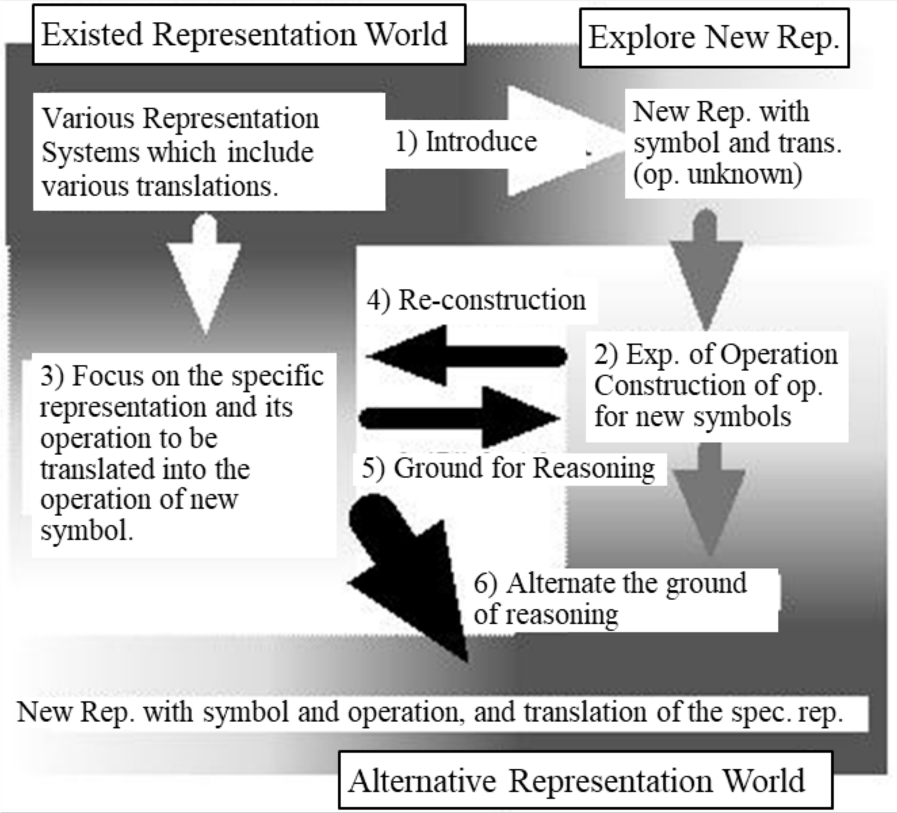


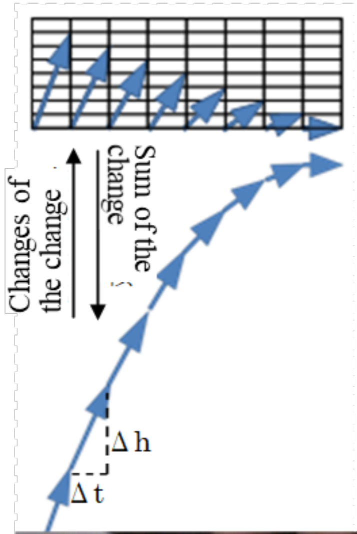
Time-distance

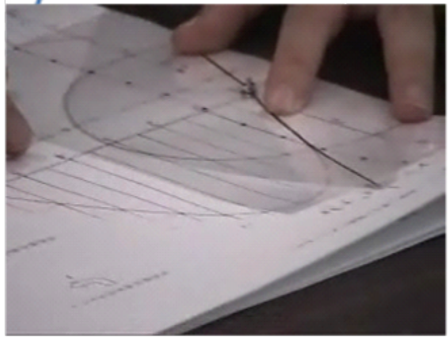
Time-velocity





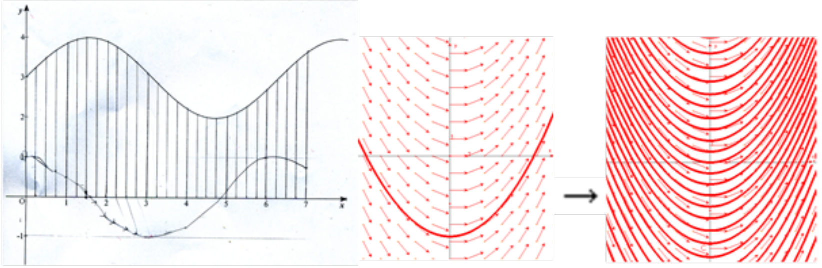




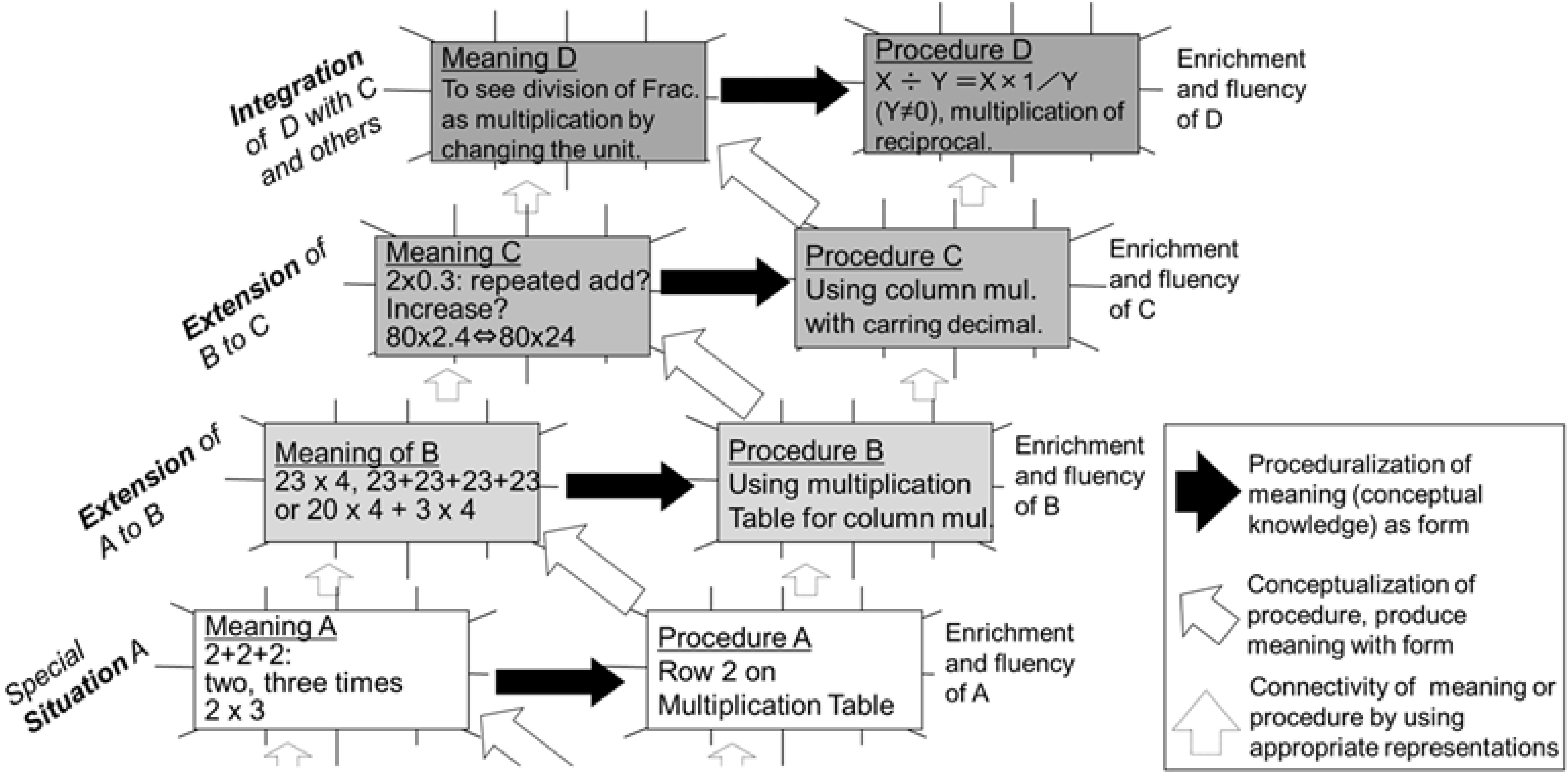


Conceptualization is discussed under the translation and operation on the graph is produced.

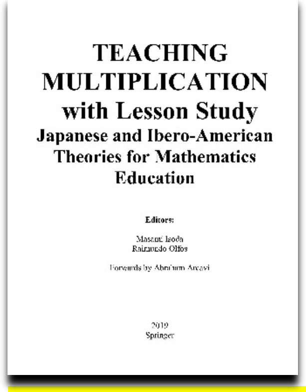
Proceduralization is not discussed here.



Conceptualization and Proceduralization



B to C and D to E:
Tape Diagram and Proportional Number Lines



A to B:
Base Ten Model by Blocks

Models for Distributions