Teachers' mathematics content knowledge about the meaning of fractions

Hongki Julie

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3. What is The Meaning of a Fraction

Introduction

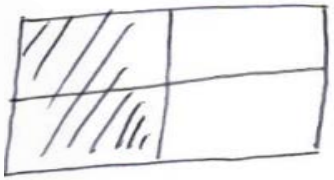
- Lortie-Forgues, Tian and Siegel (2015): one that influences students 'success in studying mathematics, physics, chemistry, and biology is students' understanding of fractions and decimals [1].
- Lortie-Forgues, Tian and Siegle (2015) and MA (1999): many students have difficulty in understanding the meaning of a fraction [1, 2].
- Ma (1999): the difficulty of understanding the meaning of fractions is not only experienced by students when they learn about fractions, **but also** experienced by **teachers** when they teach about fractions.

Questions

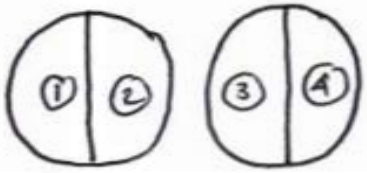
(a) what are the meanings of $\frac{2}{4}$?

(b) how to represent $\frac{2}{4}$?

How about teachers' understanding about the meaning of fractions?

The meaning of $\frac{2}{4}$	The representation of $\frac{2}{4}$	The number of teachers and the percentage	Consistency between the meaning and the representation
Two parts of the four whole parts.		15 = 53.57 %	Consistent.
A ratio between two and four.		8 = 28.57 %	Inconsistent, because in the representation made by teachers, fraction $\frac{2}{4}$ was interpreted as two parts of the for whole parts.

How about teachers' understanding about the meaning of fractions?

The meaning of $\frac{2}{4}$	The representation of $\frac{2}{4}$	The number of teachers and the percentage	Consistency between the meaning and the representation
Two divided by four.		2 = 7.14 %	Inconsistent, because in the representation made by teachers, fraction $\frac{2}{4}$ was interpreted as two parts of the for whole parts..
Two divided by four.		2 =7.14 %	Consistent.

Lamon (2001, in Ayunika, 2012)

- The meaning of fractions in the teaching-learning process was a complex process because the concept of fraction had a number of interpretations, namely (a) fraction as a part of the whole, (b) fraction as the result of a measurement, (c) fraction as an operator, (d) fraction as a quotient, and (e) fraction as a ratio [3].

$\frac{2}{4}$ can be interpreted and represented

a. Two parts of the four whole parts is represented as follows:

b. The results of measurements, for example, someone measures the length of an object, then $\frac{2}{4}$ is defined as the length of the object measured.

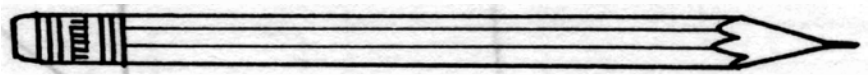
It means the length of the object is $\frac{2}{4}$ part of the length of the instrument used.

For example, students carry out activities of measuring the length of objects using paper cuttings, where it is assumed that the length of the piece of paper is one unit.

Determine the length of the following pencils!

Therefore, $\frac{2}{4}$ can be interpreted and represented as follows:

$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	**1**



$\frac{2}{4}$ can be interpreted and represented

c. $\frac{2}{4}$ could also be interpreted as $\frac{2}{4}$ part of a set of objects or of a size. For example: $\frac{2}{4}$ part of 40 students, $\frac{2}{4}$ part of 4 kg of sugar, $\frac{2}{4}$ hour, $\frac{2}{4}$meter. In this case, a fraction is interpreted as an operator.

d. $\frac{2}{4}$ could also be interpreted as a result of two divided by four.

For example, Mrs. Helen has 2 kg of sugar. The sugar is packaged into 4 smaller packages, and the weight of the sugar in each package can be expressed by $\frac{2}{4}$ kg.

Another example, Aji has 2 pieces of bread. He divided the bread to four of his friends, so the part of bread received by each friend of Aji could be stated by $\frac{2}{4}$.

$\frac{2}{4}$ can be interpreted and represented

e. $\frac{2}{4}$ could also be interpreted as a ratio.

For example a fish pond has a width of 2 meters and a length of 4 meters, then the ratio between the width and length of the fish colan can be expressed by $\frac{2}{4}$. Another example, Mr Wawan's land area is 2 ha, while Mrs. Tika's land area of is 4 ha, then the ratio of Mr Wawan's land area and Mrs. Tika's land area can be expressed with fractions $\frac{2}{4}$.

Sutarto Hadi (2002)

- Developed a professional development model for junior high school teacher to introduce a realistic mathematics education (RME) approach.
- The model was developed in his study were as follows:
  1. conducting workshops for junior high school teachers who would be the subject of the research
  2. conducting classroom practice
  3. to making a reflection.
- It was a good model for the professional development of mathematics teachers in Indonesia, in particular to introduce a new approach in teaching mathematics [4, 5].

There are four main principles in the RME

1. **Guided reinvention;**
   - Students were given the experience "like" the mathematicians to find it, but in the reinvention process was done by the students, there were teacher guidances.
   - So, in the reinvention process, there needs to be a student communication, and there was a negotiation process between one student and other students.
   - The communication and negotiation process between one student and other students were intended to develop students' findings gradually until the students can achieve the mathematics formal knowledge.

There are four main principles in the RME

2. **The progressive mathematizing;**
   - Students learned to construct a formal mathematical knowledge through to solve the contextual problem series with varied context.
   - Through the process of solving a series of contextual problems, students were expected to improve
     a. the ability to translate contextual problem sentences to mathematical sentences ranging from simple to complex mathematical sentences,
     b. the ability to solve the problems ranging from the informal to the formal, and
     c. the ability to retranslate problem solutions in mathematical sentences into the context of the problem.

There are four main principles in the RME

3. **Didactical phenomenology;**
   - The students were given the opportunity to explore phenomena or situation series that could make students experience a progressive mathematization.

There are four main principles in the RME

4. **Self-developed models.**
   - Models were interpreted as a representation of translating problems into the mathematics language and problem solving in the problem solving stages.
   - Students are given the opportunity and are helped to express their ideas in representing sentences of contextual problems into mathematical sentences ranging from informal to formal mathematics sentences and solving the problem ranging from informal strategies to formal strategies.

Research Methodology

- The type of research: the Cobb and Gravemeijer design research model.
- The approach to the learning model that would be used by the researcher in providing workshops is the RME approach.
- The workshop was conducted in 5 hours of lessons.
- Workshop material included (1) the meaning of fractions as a part of the whole parts and how to represent it, (2) the meaning of fractions as a divided process and how to represent it, and (3) the meaning of fractions as ratio and how to represent it.

Research Methodology

- The workshop material was only limited to three fraction meanings according to Lamon, due to the limited time given by the PPG program manager to the researcher.
- The research subjects: 29 junior high school mathematics teachers
- The research methods: making a field note and a test.
- The instrument: the test sheet.
- The test consists of two questions and the teacher's time to do the test is 20 minutes.
- The test took place after the workshop was completed.
- The data analysis technique: reducing data, presenting data, and making conclusions [12].

The First Problem

- Edi has one box of brownies. The cake was divided equally to four of his friends, namely Karyo, Susan, Surya, and Kristin.
  1) What part of the cake does Susan get?
  2) State the process and the result of division of Edi's brownies in mathematical symbols!
  3) What fraction meanings do you get from the answers to questions one and two?

The Second Problem

- Sam has 3 boxes of cheese brownies. The cake was distributed to Sidik, Mira, Erna, and Tasya.
  1) Describe how to divide Sam's brownies cake!
  2) State the process and results of Sam's brownies cake in a mathematical symbol!

  Because Tasya did not like the brownies cake, she gave 2 pieces of what she earned to Mira.
  3) What is the ratio of the number of pieces of Tasya's and Mira's brownies?
  4) What is the ratio of the parts of Tasya's and Mira's brownies?
  5) What do the meanings of fractions that you get from question one to four?

The Third Problem

- Surti has 2 boxes of brown chocolate chips. The cake was shared with John, Mimi, and Erna.
  1) Describe how to divide Surti's brownies cake!
  2) State the process and results of Surti's brownies cake in a mathematical symbol!

  Because John would share the cake that he got with his three younger sisters, then Mimi gave one piece of brownies that she got to John so that each of John's younger sisters got one piece of brownies.
  3) What is the ratio of the number of pieces of Mimi’s and Jhon’s brownies?
  4) What is the ratio of the parts of Mimi’s and Jhon’s brownies?
  5) What do the meanings of fractions that you get from question one to four?

Questions of a test

1. What are the meanings of the $\frac{3}{5}$ fraction?
2. Please, make a representation for fractions $\frac{3}{5}$!

The Results of The Test

No.	The number of the meaning of $\frac{3}{5}$ which made by the teacher	The number of the representation of $\frac{3}{5}$ which made by the teacher	The number of the representation of the $\frac{3}{5}$ fractions which consistent with the meaning of fractions $\frac{3}{5}$	The number of the teacher	Percentage (%)
1.	2	1	1	1	3,45
2.	2	2	1	1	3,45
3.	2	1	1	5	17,24
4.	3	3	3	13	44,83
5.	3	1	1	2	6,9
6.	4	1	1	2	6,9
7.	4	4	4	2	6,9
8.	4	2	2	2	3,45
9.	3	2	2	1	3,45

Conclusions

a. Before the teacher participated in the workshop, the conditions of the teachers were as follows: (1) there were two the meanings of $\frac{2}{4}$ which could be interpreted and represented correctly by teachers, namely (a) two parts of the four whole parts which made by 15 of 28 teachers or 53.57% and (b) two divided by four which made by 2 of 28 teachers or 7.14%; (2) 26 of 28 teachers or 92.86% represented the meaning of $\frac{2}{4}$ as two parts of four whole parts or 11 of 28 or 39.29% who represented the meaning of $\frac{2}{4}$ as two parts of the four whole parts, but they were not interpret fractions $\frac{2}{4}$ as two parts of the four whole parts.

Conclusions

b. After the teacher participated in the workshop, the conditions of the teachers were as follows: (1) there were five meanings and representations of $\frac{3}{5}$ that the teacher was able to make correctly, i.e. (a) three parts of the five whole parts by all teachers, (b) three divided by five by 16 of 29 teachers or 55.17%, (c) a ratio between three and five by 15 of 29 teachers or 51.72%, (d) $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$ part of one whole part by 2 of 29 teachers or 6.90%, and (e) $\frac{3}{15}$ part from three whole parts by 2 of 29 teachers or 6.90%, and (2) 51.72% the teacher was able to explain two meanings of $\frac{3}{5}$ and represent meaning correctly.