AN ANALYSIS OF REPRESENTATION FORMS IN LEARNING MATHEMATICS ON THE TOPIC OF CUBOID'S VOLUME

Tea Tasia Wiwin[1,a)] and Yustina Mogi[2,b)]

[1,2] *Department of Mathematics Education, Faculty of Teacher Training and Education, Sanata Dharma University, Mrican, Tromol Pos 29, Yogyakarta 55002, INDONESIA*

[a)]teatasia@gmail.com
[b)]yustinakatharina@gmail.com

Abstract

This study aims to determine the forms of representation that students use in learning mathematics on the topic of cuboid's volume. The method used in this study is descriptive qualitative. Subjects in this study were 8th grade students of SMP Negeri 2 Purwosari which amounted to 31 students. The forms of data in this study are student worksheets and interviews. Based on the results of the study, the forms of representation that students use in learning mathematics on the topic of cuboid's volume are external representation and Bruner's representation. External representation consists of concrete objects representation and verbal representation. Meanwhile, Bruner's representation consists of iconic and symbolic representations.

Keywords: analysis, representation forms, cuboid's volume.

Introduction

Mathematics is one of the disciplines that can improve thinking ability and argue, contribute in solving everyday problems and in the world of work, and provide support in the development of science and technology (Susanto, 2013: 185). Learning mathematics is a sufficient condition to continue education to the next level. Because by learning math, we will learn reasoning critically, creatively, and actively. Mathematics is an abstract idea that contains symbols, then mathematical concepts must be understood first before manipulating the symbols. However, in general the mathematical abstraction, relatively not easy to understand by students.

Therefore, it takes something that can help the student to communicate the abstract mathematical concepts. One means to communicate these ideas is a mathematical representation. According to NCTM (Sabirin, 2014) representations are expressions of mathematical ideas or

ideas that students display in their quest to find a solution to the problem they are facing. With representation, problems that all look difficult and complicated can be viewed more easily and simply, so the problems presented can be solved more easily. Thus representation is important in the learning of mathematics.

Based on the above description, it is necessary to study about the forms of representation used by students in learning mathematics. This study aims to analyze the forms of representation in learning mathematics.

Understanding of Mathematical Representation

Representation is defined as a new form of translations of a problem or idea, or translation of a diagram from the physical model into symbols or words (Susanto, 2013: 216). Furthermore, Susanto explains that representation can help the child explain the concept or idea and make it easier for children to get the solving strategy. In addition, it can increase the flexibility in answering math problems. According to NCTM (in Sabirin, 2014) representations are expressions of mathematical ideas or ideas that students display in their quest to find a solution to the problem they are facing. Representation is the way one uses to communicate answers or mathematical ideas (Cai, Lane, & Jacabcsin in Syarifah Fadillah, 2008).

Lestari & Yudhanegara (2015: 83) explains that the ability of mathematical representation is the ability to present symbols, tables, graphs, charts, equations, or other mathematical expressions in other forms. From some of the above opinions it can be concluded that representations are expressions of mathematical ideas that students display as models or forms of substitution of a problem situation that is used to find the solution of the problem it is facing.

Forms of Mathematical Representation

Hiebert and Carpenter (in Syarifah Fadillah, 2008) argue that basically representations can be expressed as internal representations and external representations. Thinking of a mathematical idea that is then communicated requires an external representation of its form: verbal, drawing and concrete objects. Pape and Tchoshanov (2001) argue that there exists a mutually interrelated relationship between internal representation and external representation. The relationship between these internal and external representations can be illustrated as in the following figure:

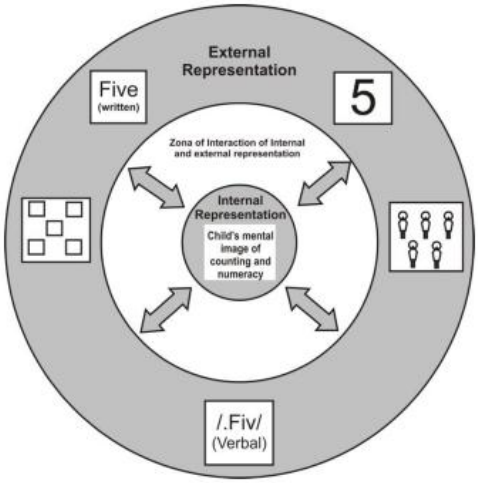

Figure 1. Relationship between Internal and External Representation

Bruner (Luitel, in Wiryanto, 2014), distinguishes three types of mental representational models, namely enactive representations, iconic representations, and symbolic representations:

a. Enactive representation is a motor sensory representation formed by action or movement. At this stage the representations made through the actions of the child are directly involved in manipulating (tinkering) objects. At this stage the child learns a knowledge where knowledge is actively learned by using concrete objects or using real situations, and children without the use of imagination or words. He will understand something from doing or doing something.

b. Iconic representation relates to image or perception, which is a learning stage where knowledge is represented in visual images, verbal visuals, drawings, or diagrams depicting concrete activities or concrete situations in the enactive phase. Language becomes more important as a medium of thought.

c. Symbolic representation deals with the language of mathematics and symbols. The child is no longer associated with objects as in the previous stage. Children are able to use notation without dependence on real objects. At this symbolic stage, learning is represented in the form of abstract symbols, the arbitral symbols used by agreement in the relevant field, both verbal symbols, mathematical symbols and other abstract symbols.

Methodology

The type of this study is descriptive research with qualitative approach. The study was conducted on Monday, March 13, 2017 at SMP Negeri 2 Purwosari. Subjects in this study were 8^{th} grade students, amounting to 31 students. Analysis to see the emerging forms of representation performed on each activity. In first and second activity, the analysis was conducted thoroughly for the subjects in the class of 31 students who had been divided into 5 groups. Meanwhile, on third activity analysis is only done on 5 subjects. These five subjects are selected based on students' activeness during the learning process. The object of this study is the forms of representation in mathematics learning on the topic of cuboid's volume.

The main instrument in this study is researcher and its supporting instrument are Hypothetical Learning Trajectory (HLT) and Student Worksheet. Technique of collecting data in this study are participant observation and test. Data obtained from the implementation of learning and tests were analyzed. This analysis aims to describe the forms of representation that appear in the learning process.

Results and Discussion

The topic constructed in this study is cuboid's volume. The core activities in the learning process are divided into 3 activities as follows:

a. Activity 1

In the first activity, students are asked to explain the following issues:

There are two cuboids containing 40 unit cubes and 300 unit cubes.

1. Determine the number of unit cubes that fill the cuboids!
2. Explain how to calculate it!
3. Describe the relationship with the length, width and height of the cuboids.

b. Activity 2

In second activity, students are asked to explain the following issues:

Miss Tini has 12 unit cubes.

1. Determine the size, width, and height of the cuboids that may be formed from those cubes!
2. Explain your reasons!

c. Activity 3

In the third activity, students are asked to solve two questions. The questions are given to measure the extent to which students' understanding of the studied volume of the cuboids has been studied. The two questions are:

1. Adi has a tub shaped with a length of 9 m, width 7 m, and height 5 m. The tub will be filled with water. How much water does it take to fill 2/3 of Adi's tub? (Express in liter!)

2. Miss Anisa wants to make a storage area of rice in the form of a cuboid. If she wants the storage area to hold 30 liters of rice, what is the height of the storage area if the length and width are 50 cm and 20 cm respectively?

Activity 1

The first activity aims to direct students to discover the concept of cuboid's volume. In this activity, each group is asked to determine the number of unit cubes that fill a cuboid. The researchers provided two blocks each containing 40 unit cubes and 300 unit cubes. The first cuboid containing 40 unit cubes is used interchangeably by groups 1, 2, 3, and 5. While the second cuboid containing 300 unit cubes is used by group 4. The following representation forms appear in the first activity:

1. External Representation
    a. Concrete Objects

        Students are directly involved in manipulating objects in the form of cuboids containing a number of unit cubes. Students use concrete objects in the form of cuboids containing a number of unit cubes to build the concept of cuboid's volume.

Figure 2. Students observe cuboid containing 300 unit cubes

b. Verbal

Students explain how they determine the number of unit cubes that fill the cuboid that they had previously held. Students also explain the relationship between the way they calculate the length, width and height of the cuboid.

Figure 3. Students explain how to determine the number of unit cubes

2. Bruner's Representation

a. Enactive Representation

Students are directly involved in manipulating objects in the form of cuboids containing a number of unit cubes. Students use concrete objects in the form of cuboids containing a number of unit cubes to build the concept of cuboid's volume.

Figures 4 & 5. Students observe cuboids containing 40 unit cubes

b. Symbolic Representation

i. Mathematical Statements

Symbolic representation of mathematical statements appears in the first activity. Students can explain the relationship between the way they calculate the length, width and height of the cuboid. The following interaction of researchers and students represented by group 5:

R: "How do you determine the number of unit cubes that fill this cuboid?"

S: "The first step is to determine the number of unit cubes contained at the top level."

R: "What is the number of unit cubes at the top level?"

S: "The number of unit cubes at the top level is 20 and is obtained from 5 × 4."

R:"How many unit cubes that filled this cuboid?"

S: "The number of unit cubes that meet this block is 40 obtained from 20 + 20 or 2 × 20."

R: "Why 20 + 20 or 2 × 20?"

S: "Because there are 2 levels."

R: "What kind of flat builds that formed by the unit cubes at the top level?"

S: "Rectangle."

R: "Then what does 20 represent on a rectangle?"

S: "The area of the rectangle?"

R: "The area of the rectangle was obtained from?"

S: "Area is obtained from length × width."

R: "The number of levels actually represents what?"

S: "High."

R: "So to calculate the number of unit cubes that meet this cuboid can be done with?"

S: "Multiplying the number of levels by the number of unit cubes at each level, where many levels represent the height and number of unit cubes at each level representing the area of the rectangle (length × width).

ii. Mathematical Symbols

Symbolic representation occurs when researchers ask students to write down how they determine the number of unit cubes that fill the cuboids. Students use mathematical symbols related to volume cuboid. Visible students use $V, p, l,$ and t as symbols for volume, length, width, and height.

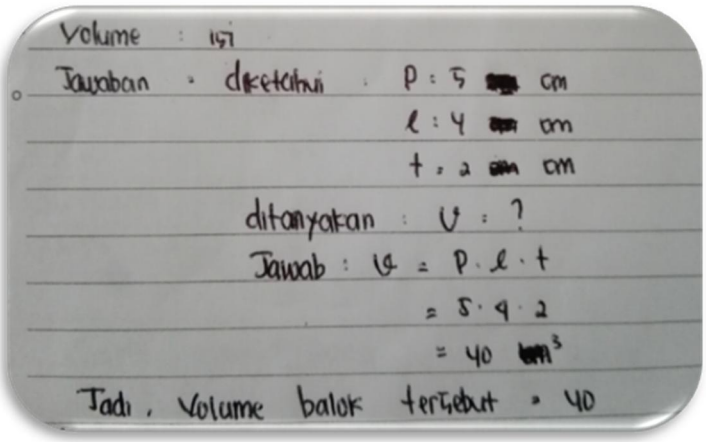

Figure 6. Results of one group's work for the first activity

Activity 2

1. External Representation

   a. Concrete Objects

   Students directly involved in manipulating (tinkering) objects in the form of unit cubes numbered 12 units. Students use concrete objects in the form of unit cubes to determine the size, width, and height of the cuboids that may be formed from the 12 unit cubes.


Figures 7 & 8. Students form cuboid of 12 unit cubes

b. Verbal

After direct involvement in manipulating cuboids containing a number of unit cubes, students explain how they determine the length, width, and height of the cuboid that may be formed from the 12 unit cubes.

2. Bruner's Representation

a. Enactive Representation

Students directly involved in manipulating (tinkering) objects in the form of 12 unit cubes. Students use concrete objects in the form of unit cubes to determine the size, width, and height of the cuboids that may be formed from the 12 unit cubes.


Figure 9 & 10. Students form cuboid of 12 unit cubes.

b. Symbolic Representation

i. Mathematical Statements

Symbolic representations of mathematical statements appear in the second activity. Students can use the concepts of factor to determine the length, width, and height of the blocks they will form from the 12 unit cubes. The following interaction of researchers and students represented by group 3:

R: "Have you formed a block?"

S: "It's Miss. We have 3 forms of cuboids."

R: "How do you determine the length, width, and height of the cuboid?"

S: "By trial and error. Because the volume is 12 then we have to find 3 numbers which when multiplied the result is 12. Well, the three numbers each represent the length, width, and height. "

R: "There is no relationship between the numbers with 12?"

S: "Relationship? What do you mean? "

R: "You guys say that 3 numbers are multiplied by 12. Which means those numbers are what 12?"

S: "Oh, a factor of 12 Miss."

P: "Then to make it easier, what do you do now?"

S: "Registering factors of 12 Miss."

P: "Good. What are the 12 factors? "

S: "1,2,3,4,6, and 12?"

P: "Okay, now please specify the length, width, and height using the combination of the numbers?"

ii. Mathematical Symbols

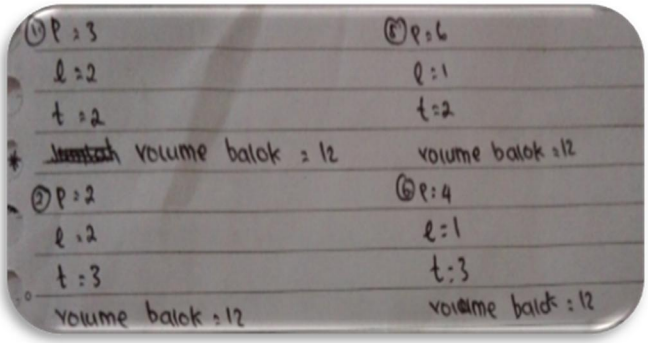

Figure 11. Result of work of one group for the second activity

Symbolic representation occurs when the researcher asks students to write down the number of cuboids they can shape by using the 12 unit cubes distributed to them. Students use mathematical symbols related to cuboid's volume. Visible students use $V, p, l,$ and t as symbols for volume, length, width, and height.

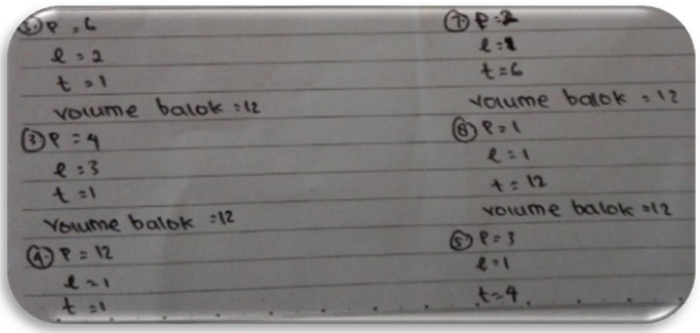

Figure 12. Result of work of one group for the second activity

Activity 3

1. Bruner's Representation

   Bruner's representation that appears in the third activity is a symbolic representation. S1, S2, S3, S4, and S5 use mathematical symbols relating to volume to represent elements that are known and asked in question. S1, S2, S3, S4, and S5 using $V, p, l,$ and t as

symbols for volume, length, width, and height. Broadly speaking, the results of the work of the five subjects have similarities in the process of completion. Therefore, researchers only show S5 work only.

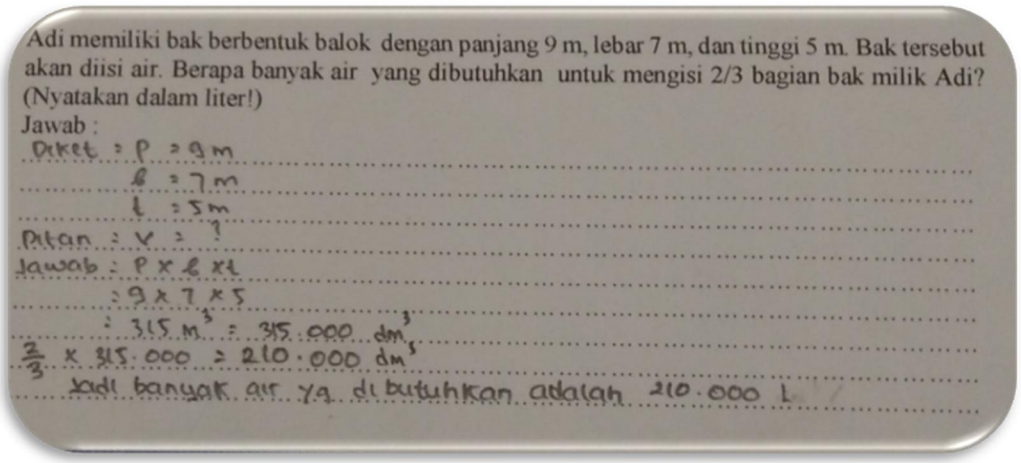
Adi memiliki bak berbentuk balok dengan panjang 9 m, lebar 7 m, dan tinggi 5 m. Bak tersebut akan diisi air. Berapa banyak air yang dibutuhkan untuk mengisi 2/3 bagian bak milik Adi? (Nyatakan dalam liter!)
Jawab :
Diket : p = 9 m
l = 7 m
t = 5 m
Ditan : V = ?
Jawab : p x l x t
= 9 x 7 x 5
= 315 m^3 = 315.000 dm^3
$\frac{2}{3}$ x 315.000 = 210.000 dm^3
Jadi banyak air yg dibutuhkan adalah 210.000 L

Figure 13. Result of S5's work for question number 1 on third activity

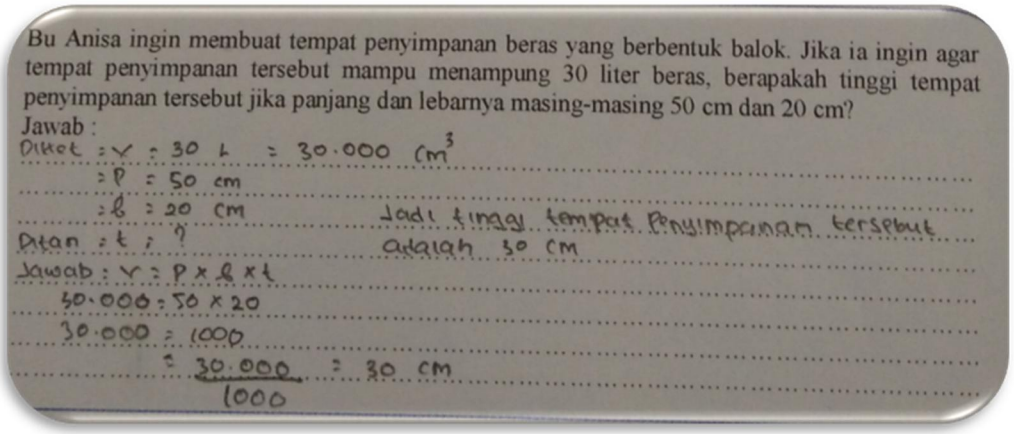
Bu Anisa ingin membuat tempat penyimpanan beras yang berbentuk balok. Jika ia ingin agar tempat penyimpanan tersebut mampu menampung 30 liter beras, berapakah tinggi tempat penyimpanan tersebut jika panjang dan lebarnya masing-masing 50 cm dan 20 cm?
Jawab :
Diket : V = 30 L = 30.000 cm^3
: p = 50 cm
: l = 20 cm
Ditan : t = ?
Jawab : V = p x l x t
30.000 = 50 x 20
30.000 = 1000
t = 30.000 / 1000 = 30 cm
Jadi tinggi tempat penyimpanan tersebut adalah 30 cm

Figure 14. Result of S5's work for question 2 on third activity

Conclusion

Based on the data analysis can be concluded that the forms of representation that appear in the learning of mathematics on the topic of the cuboid's volume is an external representation, enactive representation, and symbolic. In the first and second activity the emerging form of representation is an external, enactive, and symbolic representation. While the activity of the 3 forms of representation that emerges is a symbolic representation only. In this study, the form of internal representation does not appear. This is because researchers do

not conduct in-depth interviews on the subject of research. Therefore, researchers who will conduct similar research should conduct in-depth interviews and prepare test questions that can elicit various forms of representation.

References

Fadillah, Syarifah. 2008. *Menumbuhkan Kemampuan Pemecahan Masalah Dan Representasi Matematika Melalui Pembelajaran Open Ended.*

Lestari, Eka Kurnia & Yudhanegara, Rdwan Mokhammad. 2015. *Penelitian Pendidikan Matematika.* Karawang: Replika Aditama.

Sabirin, Muhamad. 2014. *Representasi dalam Pembelajaran Matematika.* JPM IAIN Antasar Vol. 01 No. 2 January – June 2014, p. 33-34.

Susanto, Ahmad. 2013. *Teori Belajar dan Pembelajaran di Sekolah Dasar*. Jakarta:Prenadamedia Grup.

Wiryanto. 2014. *Representasi Siswa Sekolah Dasar dalam Pemahaman Konsep Pecahan.* Jurnal pendidikan teknik elektro. Vol. 03 No. 03 tahun 2014, p: 593-603.