ANALYSIS OF MATHEMATICAL CONNECTION AND COMMUNICATION TOPIC OF THE RELATION OF CENTRAL ANGLE AND INSCRIBED ANGLE IN A CIRCLE IN GRADE VIII

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Abstract

This research aims to investigate mathematical connection and communication of students grade VIII SMP Pangudi Luhur 1 Yogyakarta in topic the relation of central angle and inscribed angle in cycle. This research was a descriptive qualitative research. The data was collected through observation, instructional video recording, the result of test, and interview of 8 subjects. Instructional video was analyzed in qualitative with making the transcript of the video, determining topics of the data, and categorizing the data. The result of the tests were analized qualitatively to determine the students' mathematical connection and communication. Result of this research showed that some students can associate of the relation of central angle and inscribed angle of a circle. Most of the students have not been able to apply the mathematical connection to solving other mathematics. This is because students do not understand the problem earlier and not accustomed to using reasoning in problem solving. Students' mathematical communication ability was already good. Students can express their ideas orally but students cannot write down the good and right idea. This is because students are not used to write down their ideas mathematically.

Keywords: mathematical connection, mathematical communication, central angle, inscribed angle.

Introduction

Mathematics connection and communication is the mathematical ability of the students. According to NCTM (2000), there are five standards that describe the relevance of mathematical understanding and mathematical competence that students should know and can do. They are understanding, knowledge and skills that students need to have include problem solving, reasoning, communication, connection, and representation.

Mathematical connections and communications are very interesting to research. Mathematical connection helps students to find the relation between contextual problem and mathematical

problem, so that students can solve contextual problems by using mathematical concepts. Students communicate how they solve the problem. They can finish and communicate their idea with drawing, graphics, writing, equation, table, etc. Their expression can be called a mathematical communication.

Based on the observation of the researcher, when the students grade VIII at Pangudi Luhur 1 Yogyakarta Junior High School were given about topic parts of circle and the relation of the central angle and inscribed angle facing the same arc of the circle, the students were unable to link prior knowledge they had to solve the problems related to the circle. Therefore, the researcher wanted to observe the mathematical connection and communication of the students. Researchers observed students' mathematical connections and communication after the researchers implemented Problem-Based Learning.

Theory

A. Mathematical Connections

Mathematical connections are connection mathematical with other lessons or other topics. There are two types of mathematical connections, they are modeling connections and mathematical connections. Modeling connections are the relationships between problem situations that arise in the real world or in other disciplines with their mathematical representation. Meanwhile, mathematical connections are relations between two equivalent representations and between the completion process of each representation.

According NCTM (2000), indicators for mathematical connection ability are: (1) Recognize and use connections among mathematical ideas; (2) Understand how

mathematical ideas interconnect and build on one another to proceduce a coherent whole; (3) Recognize and apply mathematics in contexts outside of mathematics.

De Lange (Ariyadi, 2012) divide into two, they are horizontal mathematization and vertical mathematization. Horizontal mathematics deals with the generalizing process. The process of horizontal mathematization begins with the identification of mathematical concepts based on regularities and relations found through visualization and schematization of problems.

The process of horizontal mathematization can be achieved through the following activities. (1) Identification of mathematics in a general context; (2) Schematization; (3) Formulation and visualization of the problem in various ways; (4) Search regularity and relationships; (5) Transfer the real problem into the mathematical model

Vertical mathematization is a form of formalization process in which mathematical models obtained on horizontal mathematization become the foundation in the development of more formal mathematical concepts through vertical mathematical processes. Vertical mathematical process occurs through a series of activities as well as the following stages.

The process of horizontal mathematization can be achieved through the following activities. (1) Identification of mathematics in a general context; (2) Schematization; (3) Formulation and visualization of the problem in various ways; (4) Search regularity and relationships; (5) Transfer the real problem into the mathematical model. The process of horizontal mathematization and vertical mathematization can not be directly separated into two major sections in sequence, which is the vertical mathematization process takes place after the whole process of horizontal mathematization occurs intact (as seen as figure 1).

However, the two processes of mathematization can formed step by step (as seen as figure 2).

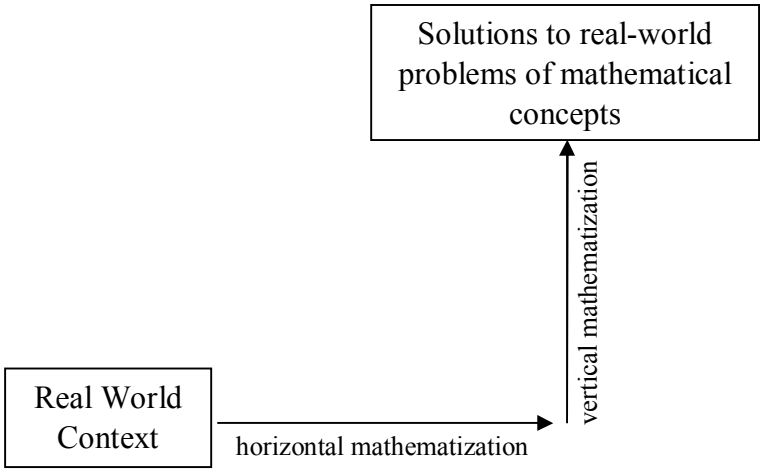


Figure 1.

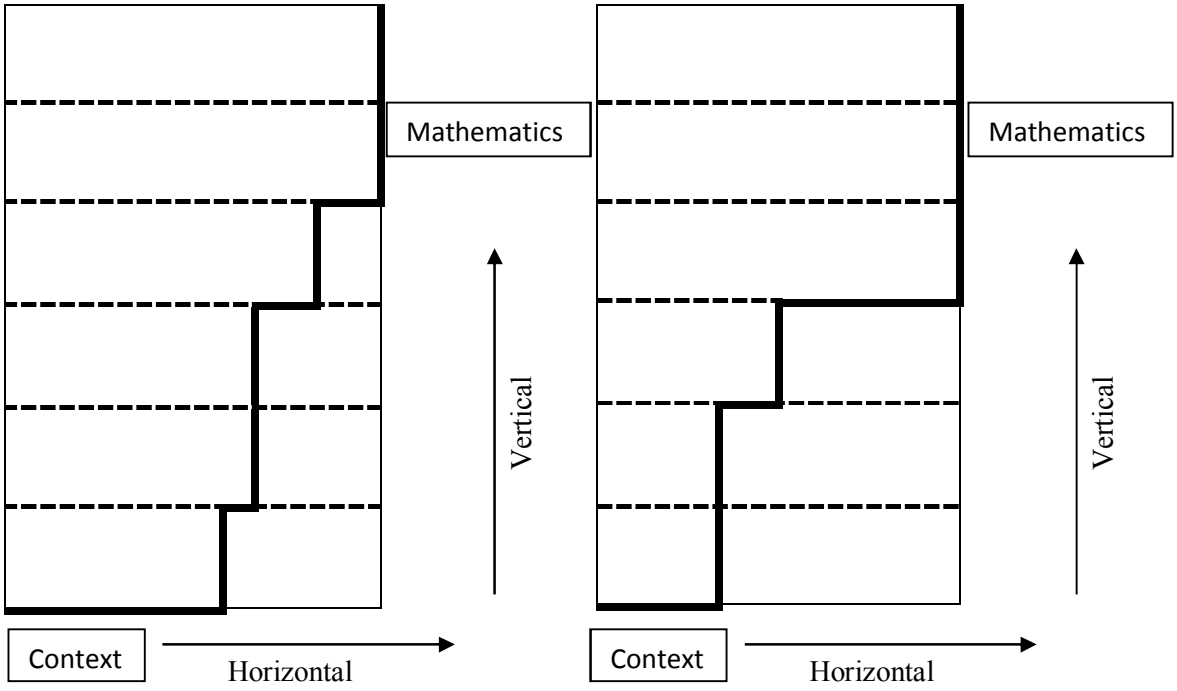


Figure 2.

B. Mathematical Communication

Mathematical communication is the ability of students to use mathematics as a tool of communication (language of mathematics) and the student's ability to communicate mathematics learned as the content of the message should be delivered (NCTM, 1989). NCTM (1989) states that students' communication skills in learning mathematics can be seen from (1) Ability to express mathematical ideas through orally, written, and demonstrate and visualize it; (2) Ability to understand, interpretation, and evaluate mathematical ideas either orally, in writing, or in other visual forms; (3) The ability to use terms, mathematical notations and structures, to present ideas, describe relationships and situational models. In classroom when students are challenged to think and reason about mathematics, communication is an essential feature as students express the results of their thinking orally and writing (NCTM, 2000).

Brenner (Prayitno, 2015) was developed a communications framework for Mathematics which summaried in table below.

Table 1. Kind of Mathematical Communication

Communication About Mathematics	Communication in Mathematics	Communication With Mathematics
Reflection on cognitive process. Description of procedures, reasoning. Metacognition-giving reasons for procedural decision	Mathemaical register Special vocabulary. Particular definitions of everyday vocabulary. Modified uses of everyday vocabulary. Syntax, phrasing. Discourse.	Problem-solving tool. Investigations. Basis for meaningful action.
Communication with others about cognition. Giving point of view. Reconciling differences.	Represention. Symbolic. Verbal Physical manipulatives. Diagrams, graphs. Geometrics.	Alternative solutions. Interpretation of arguments using mathematics. Utilization of mathematical problem-solving in conjunction with other forms of analysis.

Mathematical communication in this research is part of mathematical communication in mathematics. In this study, students are given problems about proofing the relationship of the center angle and the roving angle of the circle and the students are asked to solve it. To get the math communication is needed exploration expression mathematical vocabulary, special vocabulary or definition, representation, symbolization, numeric, graph, diagrams, verbal communication, and so on.

C. Problem-Based Learning

These are the stages of a PBL process. This process takes learners through the learning process via the following stages, which are dynamic and iterative in nature. The tutor who is the facilitator, facilitate the process (Ee, 2009). Accord to Forgarty (in Wena, 2011) stages of problem-based learning strategy are as follows:

Table 2. *Phase of Problem-Based Learning*

Phase	Behavior teacher
Phase 1: give orientation about the problem	The teacher tells the learning objectives, describes the various important logistical needs and motivates the students to engage in problem-solving activities
Phase 2 : organizing students to research	Teachers help students define and organize learning tasks related to problems.
Phase 3 : assisting independent and group investigations	Teachers encourage students to get the right information, carry out experiments, and seek explanations and solutions
Phase 4 : develop and present the results of the discussion	Teachers help students in planning and preparing appropriate instructional media, such as reports, recordings, videos, and models and helping them to convey to others.
Phase 5 : analyze and evaluate the problem-solving process	Teachers help students reflect on their investigations and the processes they use.

Methodology

The methodology of this research is descriptive qualitative approach. Descriptive research with quantitative approach is a study that aims to describe phenomena in real, where these phenomena are described based on the calculation of measure, size, or frequency (Nana Sukmadinata, 2012).

This research was conducted in SMP Pangudi Luhur 1 Yogyakarta in Academic Year 2016/2017 in class VIIIE. The data used in this research is the implementation data learning namely: (1) Transcript of video learning (2) worksheet of the implementation of lesson plan (3) Transcript of interview, and students' ability of mathematical connection and communication data which include the students' answer sheet of tests results. The data collection was conducted through test and observation. Analysis of learning implementation ftom the kind of mathematical connection and communication.

Results and Discussion

Results

Implementation of learning conducted in 2 meetings with each meeting 2x40 minutes. Learning at the first meeting aims to define the elements of the circle. While at the second meeting, students can show and prove that the measure of center angle of the circle is twice the inscribed angle that faces the same arc.

At the first meeting each student is given LKS I which contains the elements of the circle (not with the definitions). Students are asked to define each element of the circle. Previously the teacher gave instructions to draw the circle elements in the worksheet provided. From the

drawing activity students define the elements of the circle. When the student is doing the activity, the teacher walks around to accompany the students who are having difficulty.

At the second meeting, students are asked to group consisting of 3-4 students. Each group is given LKS II (student worksheet). LKS II contains some picture of the center and inscribed angle, then the students answer some questions. LKS II aims for students to show that the measure of center angle of the circle is twice as large as the inscribed angle facing the same arc. Students are given 30 minutes to discuss in groups. After the students complete the LKS I, the teacher discuss the students' work result. After students finish with LKS I, students are awarded LKS III. In LKS III students are asked to prove mathematically that the center angle of the circle is twice the inscribed angle facing the same arc. Here is an example of student work in defining the elements of a circle:

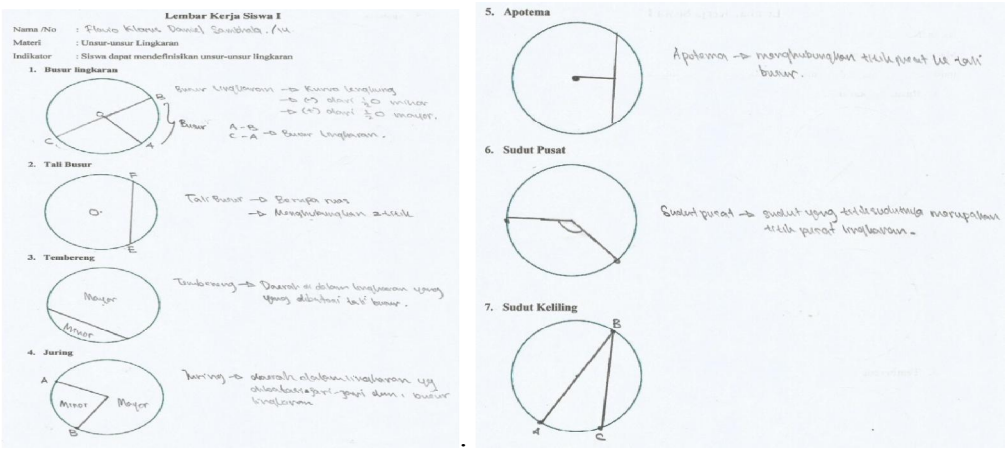


Figure 3. Sample students' solving in determine elements of circle

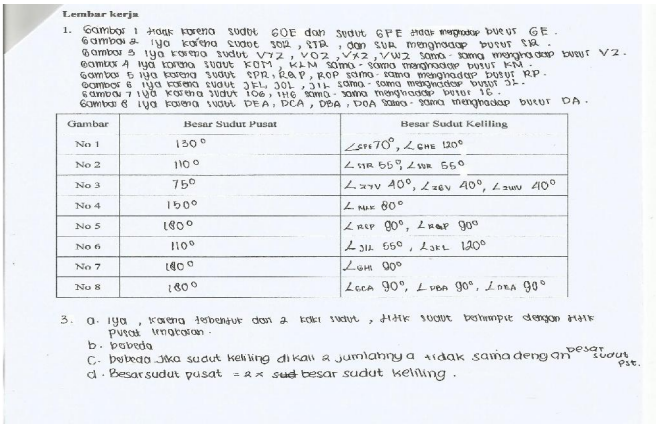
The following is an example of student work in searching for relationships between the major center angles and the circumferential angles facing the same arc.

1. Student determines three points on the circle, students name it points A, B, and C Students connect each point on the circle to the center point O. Students mention the characteristics of the circular arc, namely: curve , (-) from semi circle, and (+) from

semi circle. Students indicate the intended circular arc is the curved lines AB and CA. but student has not defined the circular arc.

2. Students determine 2 points on the circle, it is point E and F then students connect that 2 points. Students mention the characteristics of the chord but not yet complete. Students also have not defined the chord.
3. Students can draw the segment and show the major and minor segment. Students can define the segment.
4. Students determine 2 points on the circle then each point is connected to the center of the circle. Student shows major and minor sector. Students can define the sector of circle.
5. Students draw a secant then students draw a perpendicular line from the center point to the secant. Students are less precise in defining apothem.
6. Students can draw the center angle and define it.
7. Students can draw the inscribed angle but have not defined it yet.

The following is an example of student work sheet in searching for relation between the measure of center angles and the inscribed angles facing the same arc.



Lembar kerja

1. Gambar 1 tidak karena sudut GOE dan sudut GFE tidak menghadap busur GE.
Gambar 2 iya karena sudut SOR, RTR, dan SUR menghadap busur SR.
Gambar 3 iya karena sudut VYZ, VOZ, VXZ, VWZ sama-sama menghadap busur VZ.
Gambar 4 iya karena sudut KOM, KLM sama-sama menghadap busur KM.
Gambar 5 iya karena sudut QPR, RAP, ROP sama-sama menghadap busur RP.
Gambar 6 iya karena sudut JEL, JOL, JIL sama-sama menghadap busur JL.
Gambar 7 iya karena sudut TOE, THE sama-sama menghadap busur TE.
Gambar 8 iya karena sudut DEA, DCA, DBA, DOA sama-sama menghadap busur DA.

Gambar	Besar Sudut Pusat	Besar Sudut Keliling
No 1	130°	$\angle$GFE 70°, $\angle$GHE 120°
No 2	110°	$\angle$SUR 55°, $\angle$SWR 55°
No 3	75°	$\angle$XYV 40°, $\angle$XOV 40°, $\angle$XWV 40°
No 4	150°	$\angle$MLK 80°
No 5	180°	$\angle$RAP 90°, $\angle$RQP 90°
No 6	110°	$\angle$JIL 55°, $\angle$JEL 120°
No 7	180°	$\angle$GHE 90°
No 8	180°	$\angle$DCA 90°, $\angle$DBA 90°, $\angle$DEA 90°

3. a. iya, karena terbentuk dari 2 titik sudut, titik sudut bertumpu dengan titik pusat lingkaran.
b. berbeda
c. berbeda jika sudut keliling dikali 2 jumlahnya tidak sama dengan besar busur pst.
d. Besar sudut pusat = 2 × besar sudut keliling.

Figure 4. Sample students' solving in relate the central and inscribed angle in a circle

1. In figure 1, the student assumes the GOE angle and the GFE angle does not face the same (the GE arc). In other words, students assume in figure 1 there is no central angle and inscribed angle facing the same arc. Figure 2-8 students answered there is a center angle and a inscribed angle facing the same arc. Then the students mention the inscribed angle and the central angle that faces the same arc on each image. The student measures the center angle and the inscribed angle of each image but has not yet given the description of which center angle facing the same arc with the inscribed angle.
2. Not done yet.
3. In the answers c and d, the answer is not appropriate (contradiction).

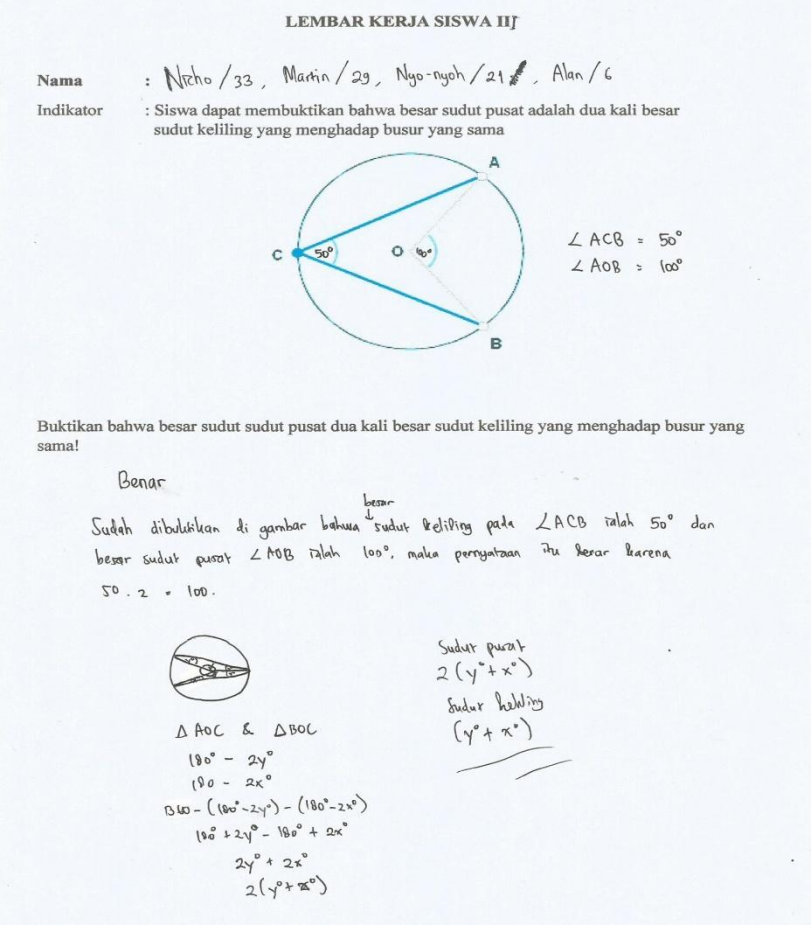


Figure 5. Sample students' solving in prove measuring of inscribed angle is double the measure of central angle

From the figure above, students measure directly the center angle and the inscribed angle. The measure of inscribed angle is double the measure of central angle so that students assume that it is already proven mathematically. After getting directions from the teacher then the students prove mathematically. Students draw a corner and center angle facing the same arc. Students draw a line from the peripheral point to the center vertex so that two equilateral triangles of the foot, the AOC triangle and the BOC triangle are obtained.

The measure of inscribed angle is $y^0 - x^0$

The measure of AOC angle is $180^0 - 2y^0$

The measure of BOC angle is $180^0 - 2x^0$

The measure of central angle equals to $360^0 - (180^0 - 2y^0) - (180^0 - 2x^0)$

So the measure of central angle is $2(y^0 - x^0)$

Discussion

A. Mathematical Connections

The mathematical connections analyzed in this journal are viewed in terms of its learning. The designed learning has taken into consideration the relation of a concept and procedure. In this learning modeling connections seen when the teacher gives instruction in the form of writing and students use the information in the form of writing into an idea for the next poured in the form of images of the elements of the circle. Furthermore, from the picture the students are again asked to assemble a definition, which in this activity students are required to use their ability in the language so that the definition of the sentence made is not ambiguous.

Modeling connections that appear are also visible when students are asked to show the relationship between the center angle and the inscribed angle at which the student was previously asked to measure the magnitude of the center angle and the inscribed angle facing and not facing the same arc. From these activities the students linked the results of the measuring activity with their ability to create a pattern and ultimately the student was able to conclude that the measure of central angle was double the measure of inscribed angle.

While the mathematical connection arises when the student activity in proving that the large center angle is twice the inscribed of the inscribed. From these activities, students give the idea of proof by using a drawing by drawing auxiliary lines so as to form an equilateral triangle of legs in a circle. Next, students use algebra to prove the magnitude of the center angle and the inscribed of the inscribed. From this activity students connect their ability in using visual representation and representation in equation form (equation).

B. Mathematical Communication

Mathematical communication analysis will be seen in terms of learning, from the observations during the learning activities, and the results of student work. In the activity of defining the elements of the circle, the teacher explores it with the students. From the activities of exploring together, spontaneous ideas emerge from students as teachers stimulate student ideas to define the elements of the circle. Some students argue about his idea of the definition of the element of the circle. Submission of these ideas is an expression of mathematical ideas through verbal, and then the teacher asks the students to write down the idea in the answer sheet so that the communication is re-expressed through writing.

In addition to drawing elements of the circle which had previously been instructed steps through the power point, students also showed the ability to understand the writing into a visual form. After the teacher confirms the exactness of the drawings that the student has made, the teacher invites the students to evaluate the naming of the circle element being drawn, such as when the student draws the shank, then that will be considered the only shield is the minor shield. Then from the students' answers, the students give guided questions so that students can finally realize that the major chord also meets the definition of the chord, as well as when the teacher asks whether the diameter includes a bow or not taliu. From this activity, students have shown their ability in evaluating mathematical ideas both orally and in writing, because after discussing it with the teacher, students then write it in their respective answer sheets.

In terms of indicating that the center angle is double the inscribed of the angle, the student has used mathematical notations, such as the use of angular notation (, the angular degree unit (o), and the ratio of the center angle are twice the inscribed angle as shown on the answer sheet one of the following students:

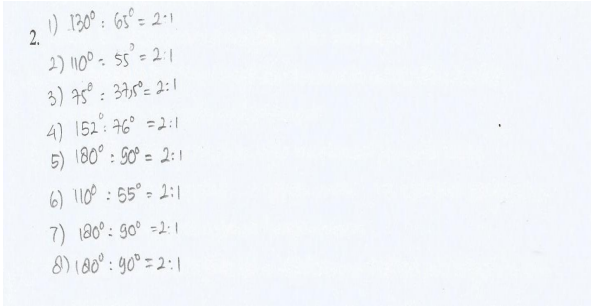


Figure 7. One of ratio notation used by student

From the figure above shows that the students use the concept of comparison by writing down the ratio of the measure center angle and the inscribed of the angle is 2: 1,

the student also uses the comparison notation (:) and concludes that the center angle is double the inscribed angle.

Conclusion

The mathematical connection that occurs in this learning is modeling connection. Modeling connections that appear are also visible when students are asked to show the relationship between the center angle and the inscribed angle at which the student was previously asked to measure the magnitude of the center angle and the inscribed angle facing and not facing the same arc. From these activities the students linked the results of the measuring activity with their ability to create a pattern and ultimately the student was able to conclude that the measure of central angle was double the measure of inscribed angle.

The mathematical communication that occurs in this learning is expression of mathematical ideas through verbal, using of equation, using of mathematical notations and expressions.

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