Mathematical Modelling Using Integer Linear Programming Approach for A Truck Rental Problem

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Abstract. Mathematics is not only about theory, but also talks about the real applications. Nowadays, mathematics is applied to solve problems in physics, economics, biology, engineering, business industries, and many more. This paper discusses a problem using mathematical modelling in one of the industrial optimisation problems, a transportation problem. However, this paper is different from other papers. Instead of solving the kind of demand-supply cases (between sources and destinations), which mostly papers discuss, this case talks about a truck rental problem, which demand-supply activities happen between sources and sources. Here, there are eight truck rentals in some areas. The problem is extracted into mathematical equations producing the objective function and some constrains. The objective function is to minimise the total transportation cost whereas the number of trucks available and the number of trucks required is the constraints. In this problem, the integer linear programming model is used to obtain an optimum solution to determine the number of trucks moved from one truck rental to another truck rental.

1. Introduction

Transportation is one of important roles in supply chain activities. Facilities, inventory, transportation, information, sourcing, and pricing contribute to improve supply chain performance in case of responsiveness and efficiency [1]. Transportation also plays a significant roles for selling price products since its logistics cost is expensive [2]. In addition, transportation is one of key factors of supply chain strategy. Transportation creates networks between producers and customers, and it depends on the transportation modes that are related to transportation costs, customer satisfaction, and efficiency [3].

There are five major modes of transport which are road, air, water, rail, and pipeline [4]. Trucks or cars, barges, trains, and pipes are some examples of road, water, rail, and pipeline transportation modes, respectively. This paper addresses road transportation which is trucks. Trucks can carry goods to urban areas or access any places [5]. Truck or road transportation modes also have the other advantages, such as having medium fuel costs and low fixed cost as well as are a favourable

transportation in terms of availability, dependability, and frequency [4]. However, there are some problems related to it.

The most common problems found are about Integrated Buyer-Seller, Integrated Production-Distribution Planning, Integrated Production-Inventory Planning, and Location-Allocation Models [6]. Gupta [7] gave some examples, such as production-distribution system, rail and urban road system, and telecommunication network. This paper highlights demand-supply one which is the same as production-distribution system or location-allocation models.

This paper starts with a literature review in Section 2. Then it will deal with the truck rental problem in Section 3, the mathematical model in Section 4, the problem solution in Section 5, and a conclusion in the last section.

2. Literature Review

Many operation research literature discuss transportation problems. Most of them focus on demand-supply cases which are divided into two types of transportation problems: a balanced and an unbalanced transportation problem.

Demand-supply cases often can be found in a considerable amount of literature. A balanced transportation problem is a problem that the total units of supply is equal to total units of demand whereas an unbalanced one is a problem that the total units of supply are less or greater than total units of demand [8]. Yadav [9] discussed a balanced distribution problem. The problem has some sources or origins and destinations with certain supplies and demands. There are also distribution or transportation costs between every origin and destination. In the same vein, Hillier and Lieberman [10] gave two examples of those two types of transportation problems. Firstly, a canned-peas company had three factories in Washington, Oregon, and Minnesota. The factories must distribute the products to four warehouses in California, Utah, South Dakota, and New Mexico. This problem is a balanced transportation case. Secondly, an airplane manufacturer having to supply aiplanes to four airline companies will produce jet engines. Those engines, later, will be installed into every jet. There are production, inventory, and transportation cost for each unit. Another case is a Powerco plant [11]. The electricity company must send power from three plants to four cities, which both facilities have the same capacities. The company must determine how much power (in Kwh) sent to satisfy their demands as well as to minimise shipment costs. Another literature, Ravindran [12], examined a balanced transportation problem in iron ore factories having two plants and three customers. Lau [13] solved a demand management problem between machine part suppliers and customers in Australia. In this problem, the distribution cost is minimised and categorised by the number of orders. In general, the given examples happened between some sources (manufacturers or plants) and some destinations (distribution centres or customers) with capacities and demands as constraints to minimise transportation costs. Mostly, the problems explained above apply a North-West Corner Algorithm which will take times to solve them.

This paper works on a demand-supply case. However, instead of solving the kind of demand-supply cases as mentioned earlier, this paper will solve a demand-supply problem happened between sources and sources.

3. A Truck Rental Problem

A rental company has a contract with a retail store to rent its trucks. The company has twenty rentals in some particular areas. A total number of trucks available are 244 trucks. Those trucks are variously distributed at twenty rentals, which every rental indicates with excess and lack trucks. That means every rental with some needed trucks that is greater than some existing trucks is called a rental with lack trucks, and, any other way, that rental is called a rental with excess trucks. These excess trucks are required to move into some rentals which needed them, and the transportation cost for moving a truck from a rental to another one is $0.5 per km.

Table 1 presents locations, demands of trucks, and availabilities of trucks at each rental. The company should be able to satisfy the demands by considering total transportation costs.

Table 1. Locations and Demands of the Rentals.

Truck Rental	Location		Trucks available	Required trucks
	x-coordinate	y-coordinate		
1	18	12	14	15
2	23	17	14	9
3	10	0	15	9
4	15	25	4	8
5	13	10	8	10
6	2	20	12	8
7	20	25	13	18
8	32	15	7	11
9	15	18	11	18
10	24	9	23	24
11	3	8	15	4
12	33	3	17	10
13	34	26	5	9
14	37	23	19	27
15	18	4	11	5
16	42	8	20	7
17	50	23	2	6
18	53	16	10	8
19	55	38	11	18
20	7	42	13	18

4. A Mathematical Model

Before commencing the mathematical model, this problem can be converted into some notations. The problem is symbolised in these indexes and notations:

Indexes:
- i : a rental with excess number of trucks.
- j : a rental with lack number of trucks.

Notations:
- c : transportation cost per km ($0.5 per km for moving a truck).
- d_{ij} : the distance between a rental and another one.
- x_{ij} : the number of trucks moved between rentals.
- s_i, s_j : the number of trucks available at a rental.
- r_i, r_j : the number of trucks required at a rental.

The next step is to classify the problem into:
- A decision variable: a rental must distribute the trucks to every rental j which needs them from every rental i which has excess trucks. Here, the number of trucks between rentals (x_{ij}) is a decision variable.
- Constraints: rentals i having excess number of trucks must move their trucks to other rentals j having lack number of trucks (constraint 1), rentals j which need trucks must accept the trucks

from other rentals i with large number of trucks (constraint 2), and the number of trucks moved from rentals i to rentals j must be nonnegative also integer (constraint 3 and 4).

• Objective: to minimise total transportation costs to move trucks between rentals.

The general mathematical model is as follows:
Objective function:

$$\min\sum_{i=1}^{n}\sum_{j=1}^{m}c \times d_{ij} \times x_{ij}$$

Constraints:

$$\forall i \in \text{excess} : \sum_{j=1}^{m} x_{ij} \leq s_i - r_i \qquad (1)$$

$$\forall j \in \text{lack} : \sum_{i=1}^{n} x_{ij} \geq s_j - r_j \qquad (2)$$

$$\forall i \in \text{excess}, \forall j \in \text{lack} : x_{ij} \in \text{integer} \qquad (3)$$

$$\forall i \in \text{excess}, \forall j \in \text{lack} : x_{ij} \geq 0 \qquad (4)$$

5. Problem Solution

Once the mathematical model is ready, initially, some rentals should be identified as having a large number of trucks and a small number of trucks.

Rental 1: $r_1 = 15$, $s_1 = 14$, lack $= 14 - 15 = -1$

Rental 2: $r_2 = 9$, $s_2 = 14$, excess $= 14 - 9 = 5$

Rental 3: $r_3 = 9$, $s_3 = 15$, excess $= 15 - 9 = 6$

Rental 4: $r_4 = 8$, $s_4 = 4$, lack $= 4 - 8 = -4$

Rental 5: $r_5 = 10$, $s_5 = 8$, lack $= 8 - 10 = -2$

Rental 6: $r_6 = 8$, $s_6 = 12$, excess $= 12 - 8 = 4$

Rental 7: $r_7 = 18$, $s_7 = 13$, lack $= 13 - 15 = -5$

Rental 8: $r_8 = 11$, $s_8 = 7$, lack $= 7 - 11 = -4$

Rental 9: $r_9 = 18$, $s_9 = 11$, lack $= 11 - 18 = -7$

Rental 10: $r_{10} = 24$, $s_{10} = 23$, lack $= 23 - 24 = -1$

Rental 11: $r_{11} = 4$, $s_{11} = 15$, excess $= 15 - 4 = 11$

Rental 12: $r_{12} = 10$, $s_{12} = 17$, excess $= 17 - 10 = 7$

Rental 13: $r_{13} = 9$, $s_{13} = 5$, lack $= 5 - 9 = -4$

Rental 14: $r_{14} = 27$, $s_{14} = 19$, lack $= 19 - 27 = -8$

Rental 15: $r_{15} = 5$, $s_{15} = 11$, excess $= 11 - 5 = 6$

Rental 16: $r_{16} = 7$, $s_{16} = 20$, excess $= 20 - 7 = 13$

Rental 17: $r_{17} = 6$, $s_{17} = 2$, lack $= 2 - 6 = -4$

Rental 18: $r_{18} = 8$, $s_{18} = 10$, excess $= 10 - 8 = 2$

Rental 19: $r_{19} = 18$, $s_{19} = 11$, lack $= 11 - 18 = -7$

Rental 20: $r_{20} = 18$, $s_{20} = 13$, lack $= 13 - 18 = -5$

From the calculations above, there are two groups: excess $= \{2, 3, 6, 11, 12, 15, 16, 18\}$ and lack $= \{1, 4, 5, 7, 8, 9, 10, 13, 14, 17, 19, 20\}$.

Secondly, the distances and transportation costs are calculated. The distances and costs between two rentals are measured with this equation (5):

$$d_{ij} = 1.3((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}, \text{ where } c = d_{ij} \times \$0.5 / km \qquad (5)$$

Table 2 shows the distances between one rental and another rental in kms.

Table 2. Distances between Two Rentals.

	1	4	5	7	8	9	10	13	14	17	19	20
2	9.19	14.71	15.87	11.11	11.99	10.48	10.48	18.48	19.80	35.96	49.76	38.59
3	18.75	33.14	13.57	35.00	34.62	24.29	21.64	46.00	46.11	59.98	76.57	54.74
6	23.26	18.11	19.33	24.29	39.54	17.10	31.98	42.32	45.67	62.52	72.77	29.33
11	20.18	27.05	13.26	31.25	38.78	20.31	27.33	46.60	48.31	64.14	78.04	44.50
12	22.74	36.95	27.55	33.22	15.65	30.46	14.06	29.93	26.51	34.12	53.74	60.93
15	10.40	27.58	10.15	27.42	23.15	18.61	10.15	35.36	34.93	48.38	65.32	51.43
16	31.63	41.48	37.79	36.14	15.87	37.43	23.44	25.61	20.55	22.10	42.50	63.43
18	45.80	50.77	52.58	44.47	27.33	49.47	38.78	27.91	22.70	9.90	28.72	68.69

In addition, a picture below illustrates the locations of the truck rentals. The calculation of the distance between to rentals uses Pythagoras' theorem, as shown in a green triangle below. In other words, the distance is the diagonal side of the triangle.
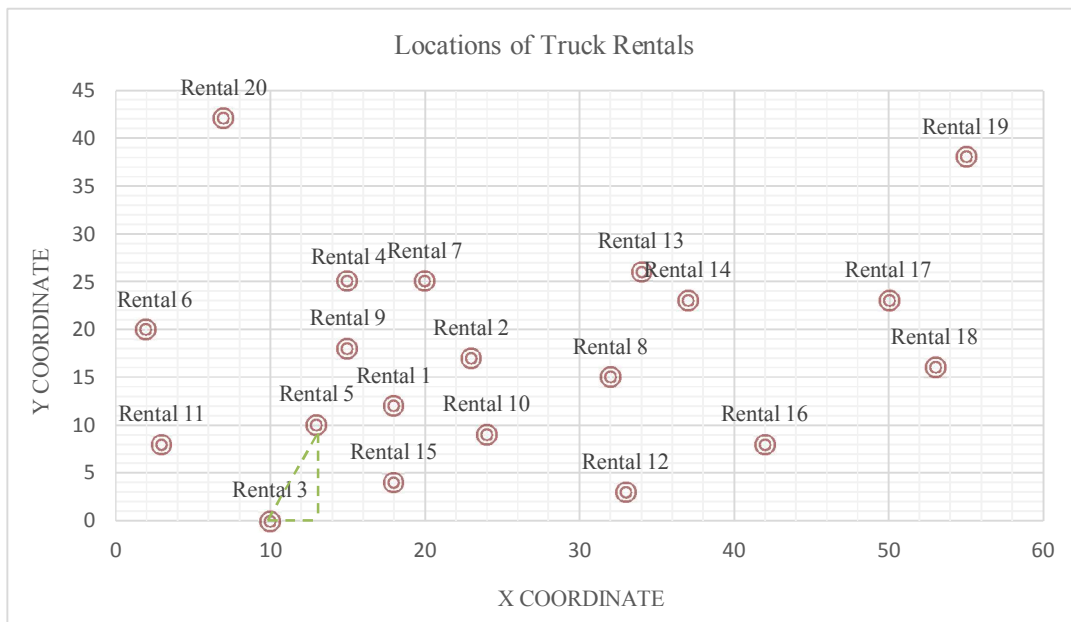
Figure 1. Locations of Truck Rentals in XY Coordinates.

Here are the mathematical formulations for a truck rental problem:

Objective function:

Minimize

$$4.60x_{21} + 7.35x_{24} + 7.93x_{25} + 5.55x_{27} + 5.99x_{28} + 5.24x_{29} + 5.24x_{210} + 9.24x_{213}$$
$$+9.90x_{214} + 17.98x_{217} + 24.88x_{219} + 19.29x_{220} + 9.37x_{31} + 16.57x_{34} + 6.79x_{35}$$
$$+17.50x_{37} + 17.31x_{38} + 12.14x_{39} + 10.82x_{310} + 23x_{313} + 23.05x_{314} + 29.99x_{317}$$
$$+38.28x_{319} + 27.37x_{320} + 11.63x_{61} + 9.05x_{64} + 9.66x_{65} + 12.14x_{67} + 19.77x_{68}$$
$$+8.55x_{69} + 15.99x_{610} + 21.16x_{613} + 22.83x_{614} + 31.26x_{617} + 36.38x_{619} + 14.66x_{620}$$
$$+10.09x_{111} + 13.53x_{114} + 6.63x_{115} + 15.63x_{117} + 19.39x_{118} + 10.15x_{119} + 13.67x_{1110}$$
$$+23.30x_{1113} + 24.16x_{1114} + 32.07x_{1117} + 39.02x_{1119} + 22.25x_{1120} + 11.37x_{121} + 18.48x_{124}$$
$$+13.77x_{125} + 16.61x_{127} + 7.83x_{128} + 15.23x_{129} + 7.03x_{1210} + 14.96x_{1213} + 13.26x_{1214}$$
$$+17.06x_{1217} + 26.87x_{1219} + 30.47x_{1220} + 5.20x_{151} + 13.79x_{154} + 5.08x_{155} + 13.71x_{157}$$
$$+11.57x_{158} + 9.31x_{159} + 5.08x_{1510} + 17.68x_{1513} + 17.47x_{1514} + 24.19x_{1517} + 32.66x_{1519}$$
$$+25.71x_{1520} + 15.82x_{161} + 20.74x_{164} + 18.89x_{165} + 18.07x_{167} + 7.93x_{168} + 18.72x_{169}$$
$$+11.72x_{1610} + 12.80x_{1613} + 10.28x_{1614} + 11.05x_{1617} + 21.25x_{1619} + 31.72x_{1620} + 22.9x_{181}$$
$$+25.38x_{184} + 26.29x_{185} + 22.23x_{187} + 13.67x_{188} + 24.73x_{189} + 19.39x_{1810} + 13.96x_{1813}$$
$$+11.35x_{1814} + 4.95x_{1817} + 14.36x_{1819} + 34.35x_{1820} \tag{6}$$

Subject to:

$$x_{21} + x_{24} + x_{25} + x_{27} + x_{28} + x_{29} + x_{210} + x_{213} + x_{214} + x_{217} + x_{219} + x_{220} \le 5 \tag{7}$$
$$x_{31} + x_{34} + x_{35} + x_{37} + x_{38} + x_{39} + x_{310} + x_{313} + x_{314} + x_{317} + x_{319} + x_{320} \le 6 \tag{8}$$
$$x_{61} + x_{64} + x_{65} + x_{67} + x_{68} + x_{69} + x_{610} + x_{613} + x_{614} + x_{617} + x_{619} + x_{620} \le 4 \tag{9}$$
$$x_{111} + x_{114} + x_{115} + x_{117} + x_{118} + x_{119} + x_{1110} + x_{1113} + x_{1114} + x_{1117} + x_{1119} + x_{1120} \le 11 \tag{10}$$
$$x_{121} + x_{124} + x_{125} + x_{127} + x_{128} + x_{129} + x_{1210} + x_{1213} + x_{1214} + x_{1217} + x_{1219} + x_{1220} \le 7 \tag{11}$$
$$x_{151} + x_{154} + x_{155} + x_{157} + x_{158} + x_{159} + x_{1510} + x_{1513} + x_{1514} + x_{1517} + x_{1519} + x_{1520} \le 6 \tag{12}$$
$$x_{161} + x_{164} + x_{165} + x_{167} + x_{168} + x_{169} + x_{1610} + x_{1613} + x_{1614} + x_{1617} + x_{1619} + x_{1620} \le 13 \tag{13}$$
$$x_{181} + x_{184} + x_{185} + x_{187} + x_{188} + x_{189} + x_{1810} + x_{1813} + x_{1814} + x_{1817} + x_{1819} + x_{1820} \le 2 \tag{14}$$
$$x_{21} + x_{31} + x_{61} + x_{111} + x_{121} + x_{151} + x_{161} + x_{181} \ge 1 \tag{15}$$
$$x_{24} + x_{34} + x_{64} + x_{114} + x_{124} + x_{154} + x_{164} + x_{184} \ge 4 \tag{16}$$
$$x_{25} + x_{35} + x_{65} + x_{115} + x_{125} + x_{155} + x_{165} + x_{185} \ge 2 \tag{17}$$
$$x_{27} + x_{37} + x_{67} + x_{117} + x_{127} + x_{157} + x_{167} + x_{187} \ge 5 \tag{18}$$
$$x_{28} + x_{38} + x_{68} + x_{118} + x_{128} + x_{158} + x_{168} + x_{188} \ge 4 \tag{19}$$
$$x_{29} + x_{39} + x_{69} + x_{119} + x_{129} + x_{159} + x_{169} + x_{189} \ge 7 \tag{20}$$
$$x_{210} + x_{310} + x_{610} + x_{1110} + x_{1210} + x_{1510} + x_{1610} + x_{1810} \ge 1 \tag{21}$$

$$x_{213} + x_{313} + x_{613} + x_{1113} + x_{1213} + x_{1513} + x_{1613} + x_{1813} \geq 4 \tag{22}$$

$$x_{214} + x_{314} + x_{614} + x_{1114} + x_{1214} + x_{1514} + x_{1614} + x_{1814} \geq 8 \tag{23}$$

$$x_{217} + x_{317} + x_{617} + x_{1117} + x_{1217} + x_{1517} + x_{1617} + x_{1817} \geq 4 \tag{24}$$

$$x_{219} + x_{319} + x_{619} + x_{1119} + x_{1219} + x_{1519} + x_{1619} + x_{1819} \geq 7 \tag{25}$$

$$x_{220} + x_{3120} + x_{620} + x_{1120} + x_{1220} + x_{1520} + x_{1620} + x_{1820} \geq 5 \tag{26}$$

$x_{21}, x_{24}, x_{25}, x_{27}, x_{28}, x_{29}, x_{210}, x_{213}, x_{214}, x_{217}, x_{219}, x_{220}, x_{31}, x_{34}, x_{35}, x_{37}, x_{38}, x_{39}, x_{310}, x_{313},$
$x_{314}, x_{317}, x_{319}, x_{320}, x_{61}, x_{64}, x_{65}, x_{67}, x_{68}, x_{69}, x_{610}, x_{613}, x_{614}, x_{617}, x_{619}, x_{620}, x_{111}, x_{114}, x_{115},$
$x_{117}, x_{118}, x_{119}, x_{1110}, x_{1113}, x_{1114}, x_{1117}, x_{1119}, x_{1120}, x_{121}, x_{124}, x_{125}, x_{127}, x_{128}, x_{129}, x_{1210}, x_{1213},$
$x_{1214}, x_{1217}, x_{1219}, x_{1220}, x_{151}, x_{154}, x_{155}, x_{157}, x_{158}, x_{159}, x_{1510}, x_{1513}, x_{1514}, x_{1517}, x_{1519}, x_{1520}, x_{161},$
$x_{164}, x_{165}, x_{167}, x_{168}, x_{169}, x_{1610}, x_{1613}, x_{1614}, x_{1617}, x_{1619}, x_{1620}, x_{181}, x_{184}, x_{185}, x_{187}, x_{188}, x_{189}, x_{1810},$
$x_{1813}, x_{1814}, x_{1817}, x_{1819}, x_{1820}$ integer $\tag{27}$

$x_{21}, x_{24}, x_{25}, x_{27}, x_{28}, x_{29}, x_{210}, x_{213}, x_{214}, x_{217}, x_{219}, x_{220}, x_{31}, x_{34}, x_{35}, x_{37}, x_{38}, x_{39}, x_{310}, x_{313},$
$x_{314}, x_{317}, x_{319}, x_{320}, x_{61}, x_{64}, x_{65}, x_{67}, x_{68}, x_{69}, x_{610}, x_{613}, x_{614}, x_{617}, x_{619}, x_{620}, x_{111}, x_{114}, x_{115},$
$x_{117}, x_{118}, x_{119}, x_{1110}, x_{1113}, x_{1114}, x_{1117}, x_{1119}, x_{1120}, x_{121}, x_{124}, x_{125}, x_{127}, x_{128}, x_{129}, x_{1210}, x_{1213},$
$x_{1214}, x_{1217}, x_{1219}, x_{1220}, x_{151}, x_{154}, x_{155}, x_{157}, x_{158}, x_{159}, x_{1510}, x_{1513}, x_{1514}, x_{1517}, x_{1519}, x_{1520}, x_{161},$
$x_{164}, x_{165}, x_{167}, x_{168}, x_{169}, x_{1610}, x_{1613}, x_{1614}, x_{1617}, x_{1619}, x_{1620}, x_{181}, x_{184}, x_{185}, x_{187}, x_{188}, x_{189}, x_{1810},$
$x_{1813}, x_{1814}, x_{1817}, x_{1819}, x_{1820} \geq 0 \tag{28}$

Following the mathematical formulations, the data are input into LINDO.

Total transportation cost is \$641.44, and a table below shows the distribution of the trucks.

Table 3. Distribution of Trucks (in Unit)

Rental	2	3	6	11	12	15	16	18	Lack
1						1			1
4				4					4
5		2							2
7	1	1				3			5
8					3	1			4
9		1		6					7
10						1			1
13	4								4
14					4		4		8
17							4		4
19							5	2	7
20			4	1					5
Excess	5	6	4	11	7	6	13	2	

It can be seen from the table the number of excess and lack trucks are not equal, or, it is called an unbalanced supply-demand transportation problem. As explained before, rental 2, 3, 6, 11, 12, 15, 16, and 18 can satisfy the demands of rental 1, 4, 5, 7, 8, 9, 10, 13, 14, 17, 19, and 20. Numbers in the table is how many trucks distributed between rentals. For instances, rental 2 supplies a truck to rental 7 and four trucks to rental 13, rental 3 supplies two trucks to rental 5 and a truck to rental 7 and 9, rental 6 supplies four trucks to rental 20, and so on. This result indicates that all supplies can fulfil all demands.

6. Conclusion

A common demand-supply problem usually happens between sources to destinations. This paper makes a mathematical model to solve a demand-supply problem happened between sources and sources. By using LINDO to get an optimum result, the total transportation cost is $641.44 with the number of iterations are 35 iterations, and all demands are satisfied. All rentals which have excess trucks distribute them to the rentals that need them.

Although the model here is still simple, this model can be extended for the future research. This model can be the complex one by adding some constrains.

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